On the logical status of superlatives

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Abstract

It is shown that superlative constructions essentially extend the expressive power of NLs since one of the readings of ambiguous superlatives, whether numeric or non-numeric, is interpreted by functions from relations to sets which are not just case extensions of type $\langle 1 \rangle$ quantifiers used to interpret "ordinary" noun phrases. In addition the size of the set of such functions is computed and compared with the size of the set of extended type $\langle 1 \rangle$ quantifiers.

1 Introduction

There is at present a large body of results concerning the semantics of comparative and superlative constructions. Such constructions and their semantics are interesting for various reasons. We know for instance that they involve quantification and comparison, that they can behave like specific anaphors and that they give rise to peculiar ambiguities when used in the object position of a sentence with a transitive verb. More recently some of these constructions or the logical elements they involve have been used in various psycholinguistic studies concerning their learnability and processing (Hackl 2009, Petroski *et al.* 2009).

In this note I am interested in some logical aspects of superlatives. First, I will discuss the origin of the ambiguity of superlatives and some constraints on their meaning. Using simple notions drived from generalised quantifier theory I will show that this ambiguity has purely logical origins: it is due to the fact that some superlatives may have more than one comparative as their source. This is well-known and has been used to explain the ambiguity of superlatives. What is less known, however, is the fact that the different comparatives from which superlatives are derived have a different logical status since they correspond to two types of formally different functions. As a consequence it will be shown that the interpretation of superlatives extends the logical power of NLs.

The following technical and notational preliminary will be useful. Since we are interested basically in comparatives and proportional quantifiers, both of which are naturally interpreted only in finite universes, we will assume that our universe of discourse E is finite. Thus all sets (of individuals) considered are sub-sets of E. For any set A, $|A|$ is the cardinality of A and for any binary relation R the set aR is defined as follows: $aR = \{x : \langle a, x \rangle \in R\}$.

Functions from sets (sub-sets of E) to truth-values are type $\langle 1 \rangle$ quantifiers. Functions from pairs of sets to truth-values or binary relations between sets are type $\langle 1, 1 \rangle$ quantifiers. They are denotations of unary determiners. Type $\langle 1, 1, 1 \rangle$ quantifiers are ternary relations between sets. They are denotations of binary determiners. In fact for some syntactic reasons, which will become obvious in a moment, we will basically be interested in two sub-classes of type $\langle 1,1,1\rangle$ quantifiers : (1) quantifiers whose type is noted $\langle\langle 1,1\rangle 1\rangle$ and (2) quantifiers whose type is noted $\langle 1\langle 1,1\rangle\rangle$. The first class corresponds to denotations of binary determiners which take two nominal arguments (as in *More students than teachers danced*) and the second class corresponds to denotations of binary determiners which take two predicative arguments (as in *More sudents danced than sang*).

We will also use the following notation for functions from sets or relations to sets or relations. A type $\langle 1,2:1\rangle$ function is a function having a set and a binary relation as argument and giving a set as result. A type $\langle 1,1,2:1\rangle$ function is a function which takes two sets and one binary relation as arguments and gives a set as a result. Some additional technical details concerning various formal results which will be used, will be given below.

Given the above terminological explanations we will consider that, strictly speaking, numerical superlatives are semantic objects corresponding to one of the above type of functions. We will consider that they are denotations of determiner-like expressions (like *the most*) and in this case they are functions of type $\langle 1,2:1\rangle$ or less often, quantifiers of type $\langle 1,1\rangle$. Sometimes we will also consider that superlatives are denotations of noun-like expressions, that is expressions obtained by the application of the determiner-like expression to a common noun. In this case they are type $\langle 2:1\rangle$ functions (or sometimes, extended or not, type $\langle 1\rangle$ functions). However I will also use more broad terminology refering to superlatives as syntactic objects denoting superlatives in the above sense, being sure that the context of use will not lead to much confusion. All the superlatives that I will consider are singular superlatives referring to one object.

A final warning. In order to express simply some ideas or to indicate some methods in a heuristic way I will use in many places of the first half of this note a mixture between natural and formal language expressions. As far as I can tell this will be not harm and will probably make various proposals clearer. This sloppiness will be avoided when discussing some linguistic applications in the final part of the paper.

In order to "derive" superlatives I will suppose that, informally, a superlative is an instance of a comparative in the sense that in the superlative a comparison with a highest number of elements (in some way specified) is made. Thus in some sense a superlative can be considered as an "abbreviation" of a multiple comparison of a conjunction of elementary comparisons in the same way as the universal quantifier (in the finite domain) can be considered, in the simplest case, as a multiple conjunction in which each conjunct is a sentence with the same predicate but whose subject varies taking all possible values within a given domain. Thus one can consider that (1) corresponds semantically to (2a) and to (2b):

(1) Leo is the tallest student.
(2a) Leo is taller than S_1 and Leo is taller than S_2 and... Leo is taller than S_n, where S_i runs over all students different from Leo
(2b) Leo is taller than any other student

The superlatives of the form given in (2b) will be called superlatives in the comparative-anaphoric form (CA form).

2 Numerical superlatives

I will start with numerical comparatives and superlatives, that is, semantically, quatifiers and more generally functions applying to sets and relations involving (numerical) comparison of cardinalities of various sets. In that way the results will be clear and at the same time directly extendable to the case of adjectival comparison. Some such non-numerical comparatives and superlatives will also be analysed and various analogies bertween two types of conpatatives and superlatives, the numerical and non-numerical ones, will be indicated.

In fact numerical comparisons make it clear that various entities can be compared in comparative constructions. To show this, let me present first the variety of numerical comparative constructions. In (3) we have comparative constructions involving what is now known as binary determiners (Keenan 1987b, Keenan and Moss 1985, Beghelli 1994, Zuber 2009):

(3a) More students than priests dance.
(3b) More students dance than talk.

In (3a) we have a binary determiner denoting a type $\langle\langle 1,1\rangle 1\rangle$ quantifier and in (3b) a binary determiner denoting a type $\langle 1\langle 1,1\rangle\rangle$ quantifier. Their semantics is given in (4a) and (4b) respectively:

(4a) $MORE(S)THAN(P)(D) = |S \cap D| > |P \cap D|$
(4b) $MORE(S)(D)THAN(T) = |S \cap D| > |S \cap T|$

We can now obtain superlatives from both those comparative quantifiers. According to the idea indicated above we have to form a multiple conjunction with similar conjuncts which differ only by one argument which varies in every conjunct. The varying argument corresponds to the complement of *than*. Such conjunctions need not to be conjunctions of sentences but may also be conjunctions of shorter gapped forms. As the equivalence between (2a) and (2b) shows such a multiple conjunction can be replaced by a shorter expression, the CA form of the superlative, in which, roughly, a quantification induced by the quantifiers $ANY\text{-}OTHER$ or $ANYTHING\text{-}ELSE$ (of the appropriate type) is involved. Suppose that students, priests, etc. are professionals denoting sets and thus they take as their values sets P_1, P_2, etc. In this case (4a) gives rise to the superlative in (5a) or in (5b):

(5a) $MOSTLY(S)(D) = MORE(S)(THAN(P_1) \wedge THAN(P_2) \wedge ...THAN(P_n)(D))$, where $P_i \neq X$
(5b) $MOSTLY(S)(D) = MORE(S)THAN\text{-}ANY\text{-}OTHER(X)(D)$

In English (5b) would correspond to something like (6) :

(6) More students than any other professionals danced.

In principle, X takes sets as values and so we have here a higher order quantification. Since we are comparing students with other professionals, the possible values of X are all sets incompatible with S. This fact and the equivalence in (7) leads to the simple semantics of $MOSTLY$ given in (8):

(7) For all sets X and Y, $|X \cap Y| > |X \cap Y'|$ iff $\forall Z(X \cap Z = \emptyset) \rightarrow |X \cap Y| > |X \cap Z|$
(8) $MOSTLY(X)(Y) = |X \cap Y| > |X' \cap Y|$

In (7) the multiple conjunction is expressed by the universal quantifier binding a second order variable. What this equivalence says informally is that instead of comparing (cardinalities) of X with all other sets incompatible with X, it is enough to compare X with its complement X'. This parallels the observation concerning the case of non-numerical superlatives: *the oldest lady*, for instance, is the lady which is older that all other ladies or it is the lady which is older that the oldest lady among all other ladies.

We can apply a quite similar reasoning to the comparative in (4b) to get the corresponding superlative. First, we have the equivalence in (9). Then we get informally from (4b) the superlative described in (10). Its formal semantics in which the equivalence in (9) is used, is given (11) :

(9) For all sets X and Y, $|X \cap Y| > |X' \cap Y|$ iff $\forall Z(X \cap Z = \emptyset) \rightarrow |X \cap Y| > |Z \cap Y|$
(10) Most S danced=More S danced that did anything else.
(11) $MOST(S)(D) = 1$ iff $|S \cap D| > |S \cap D'|$

We have thus obtained the following result: with the comparative type quantifier $MORE$ of type $\langle\langle 1,1\rangle 1\rangle$ one can associate in a unique way the superlative type $\langle 1,1\rangle$ quantifier $MOSTLY$ and with the comparative quantifier $MORE$ of type $\langle 1,\langle 1,1\rangle\rangle$ one can associate a unique type $\langle 1,1\rangle$ quantifier $MOST$. The quantifier $MOSTLY$ is not conservative. However these quantifiers are related since one is the inverse of the other (Zuber 2005) : $MOST(X)(Y) = MOSTLY(Y)(X)$. This follows from the fact that binary quantifiers from which they originate are symmetric (Zuber 2007). Observe for instance that (3a) is equivalent to (12) :

(12) More dancers are students than priests.

Thus we have two equivalent representations for $MOST$ and $MOSTLY$. As we know, there is still another representation for these quantifiers : $MOST$ can also be represented by (13a) and $MOSTLY$ by (13b) :

(13a) $MOST(X)(Y) = 1$ iff More than half of X are Y
(13b) $MOSTLY(X)(Y) = 1$ iff More than half of Y are X

These representations are logically equivalent to the previous ones given in (11) and (8) respectively. This follows from the equivalence given in (14) :

(14) For any set A and B, $|A| = |A \cap B| + |A \cap B'|$

Indeed, it follows from (14) that $|X \cap Y| > |X' \cap Y|$ is equivalent to $|X \cap Y| > |X| - |X \cap Y|$ and thus to $2 \cdot |X \cap Y| > |X|$ which is equivalent to (13a). In fact it is possible to give an infinite number of trivially different representations for the above quantifiers.

Observe that although the representation given in (5a) deriving superlatives from comparatives is in some sense most direct and natural, it is obviously not the simplest on the formal level.

The case we considered above concerns comparatives and superlatives occuring, roughly speaking, in (NPs in) subject position. Such superlatives are equal, as we have seen, to type $\langle 1, 1\rangle$ quantifiers $MOSTLY$ or $MOST$. The situation is much more complex when comparatives and superlatives occur in non-subject position since in this case a greater variety of arguments is involved and "more" objects can be compared. In this case precisely some ambiguities of superlatives can arise.

Let us see now some cases of comparatives and superlatives occuring in object position. Consider the following examples of numerical comparatives:

(15a) Leo met more students than priests .
(15b) Leo met more students than Bill.

In (15a) we have in the object position a noun phrase formed from a binary determiner denoting a comparative quantifier that we have discussed above. Its role as a direct object is to form a verb phrase with the transitive verb. One way to account formally for this fact is to consider that type $\langle 1\rangle$ quantifiers denoted by NPs in object position are (case) extended quantifiers and act as funtions from n-ary relations to n-1 ary relations whose value is determined in a precise way by their value on sets (Keenan and Westerståhl 1997). In the above case we have to do with the accusative case extension of the quantifier denoted by the noun phrase *more students than priests*. The accusative extension Q_{acc} of a type $\langle 1\rangle$ quantifier Q is defined as follows (Keenan 1987a, Keenan 1988):

(16) $Q_{acc}(R) = \{x : Q(xR) = 1\}$

Thus accusative extensions of type $\langle 1\rangle$ quantifiers applying to binary relations are specific functions from binary relations to sets. In fact they form a (proper) subalgebra of such functions. Their specificity is indicated by the following accusative extension condition AEC (Keenan and Westerståhl 1997):

(17) AEC : A function F from binary relations to sets satisfies AEC iff for any relation R and S and and any $a, b \in E$, if $aR = bS$ then $a \in F(R)$ iff $b \in F(S)$

Functions satisfying AEC are denotations of "ordinary" NPs when they occur in the direct object position. In this case they are referentially independent of any other NP.

Of course one can generalise the AEC condition to cases when an NP occurs in an other grammatical position than just subject or direct object. In this case the corresponding functions acts as arity reducers; they reduce an n-ary relation to a (n-1)- ary relation. This happens when for instance with an NP in the indirect object position of a ditransitive verb (denoting a ternary relation): an application of an NP to such a verb gives a transitive verb (denoting a binary relation (see Keenan and Westerståhl 1997 for details concerning such a generalisation).

I introduced the above remarks in order to oppose the direct object in (15a) to the "direct object" in (15b). If we suppose that *Leo* denotes l, *Bill* denotes b, *students* denotes S and *meet* denotes M, then the semantics of (15b) is given in (18a) and thus the function denoted by the "direct object" in (15b) is an instance of the function given in (18b):

(18a) $|lM \cap S| > |bM \cap S|$
(18b) $F_{A,a}(R) = \{x : |xR \cap A| > |aR \cap A|$

One can check that the function F in (18b) does not satisfy the AEC (for any non-trivial A). It is, however, argument invariant (cf. Keenan and Westerståhl 1997), that is it satisfies a weaker condition given in (19):

(19) A type $\langle 2:1 \rangle$ function F is argument invariant iff for all binary relations R and all $a, b \in E$ if $aR = bR$ then $a \in F(R)$ iff $b \in F(R)$

There are indeed various empirical differences between expressions denoting these two functions. One observes, for instance that (15b), in contradistinction to (15a), cannot be passivised. Similarly, one cannot conjoin a "real" noun phrase to the "direct object" in (15b). All this suggests that using our methods of multiple conjunctions to derive superlatives from comparatives we can get from the comparative occurring in subject position in (15b) a "new type" of superlative. And this is indeed the case: the comparative in (15b) can be extended into a CA form of superlative indicated in (20a) and whose semantics is given in (20b). The semantic superlative, that is the function $NMOST$ of type $\langle 2, 1:1 \rangle$ obtained in that way is given in (21a). If its nominal argument is fixed that we get a type $\langle 2:1 \rangle$ function given in (21b):

(20a) Leo met more students than anybody else (met)
(20b) $\forall x \neq l(|lM \cap S| > |xM \cap S|$
(21a) $F(R, X) = \{x : \forall y \neq x|xR \cap X| > |yR \cap X|$
(21b) $F_A(R) = \{x : \forall y \neq x|xR \cap A| > |yR \cap A|$

This superlative can be expressed by something like *the greatest number of* (used appropriately).

One can see that the superlative $NMOST$ is different from the accusative extension of $MOST$ and the accusative extension of $MOSTLY$. To see this it is enough to compare the formulas corresponding to their semantics. The following examples illustrate the difference :

(22a) Leo met most of the students (=Leo met more than half of students)
(22a) Leo met mostly students (Most of the persons that Leo met are students)
(22c) Leo met more students than anybody else (met students)

Thus in the semantics of (22a) the superlative $MOST$ is involved, in the semantics of (22b) the superlative $MOSTLY$ and, finally, in the semantics of (22c) the superlative $NMOST$ is involved. Let me briefly mention now some other comparative constructions from which one can derive a superlative and some from which one cannot. Consider first the example in (23) :

(23) Leo bought more books than he read.

Recall that our way of "producing" a superlative is to vary the denotation of one argument corresponding to the complement of than in a comparative construction. In the above case what should be varied is the binary relation Leo holds to books (and which should be incompatible with the relation of buying). So we get as intermediary steps something like (24a) and, given the equivalence in (9), (24b) which is equivalent to (24c):

(24a) Leo bought more books than he read, or sold, or printed or ate,...
(24b) Leo read more books than he did not read
(24c) Leo read most books

We just got the proportional quantifier $MOST$ again. We need, however, to complete this last step. Strictly speaking what we have in (22c) is an application or an instance of a type $\langle 1, 2 : 1\rangle$ or, if the argument BOOKS is fixed, of a type $\langle 2 : 1\rangle$ function F given in (25):

(25) $F(X, R) = \{y : |xR \cap X| > |yR' \cap X|\}$

So F is a function which takes two arguments : a set (in the above case BOOK) and a relation (BUY). If we fix the argument X we get a type $\langle 2 : 1\rangle$ function given in (26):

(26) $F_A(R) = \{x : |xR \cap A| > |xR' \cap A|$

So strictly speaking in (25) we do not have the type $\langle 1, 1\rangle$ quantifier we are looking for. Similarly in (26) we do not have a type $\langle 1\rangle$ quantifier which would correspond to the quantifier $MOST(A)$. We know that type $\langle 1\rangle$ quantifiers become type $\langle 2 : 1\rangle$ functions by accusative extension. They then satisfy the AEC condition. It is also true, however, that any type $\langle 2 : 1\rangle$ function which satisfies the AEC condition uniquely determines a type $\langle 1\rangle$ quantifier. Indeed the following property holds:

(27) If a type $\langle 2 : 1\rangle$ function F satisfies the AEC and the type $\langle 1\rangle$ quantifier Q is

defined as $Q(X) = 1$ iff $a \in F(\{a\} \times X)$ then $Q_{acc}(R) = F(R)$

It is easy to check that the function F_A in (26) satisfies the AEC condition (because $aR = bS$ iff $aR' = bS'$). It follows from this, from the semantics of $MOST$ and from (27) that $MOST(A)_{acc} = F_A$. This means that the comparative pattern in (24a) gives rise to the proportional superlative, denoted by an NP which can occur either in subject or in direct object position.

Let us now see the case of a comparative construction which does not give rise to a superlative. More precisely we can obtain only the comparative-anaphoric form from it and not a lexicalised superlative. Consider (28) and (29):

(28) Bill sold more cars than Leo books
(29) Bill bought more cars then Leo read books

In (28) the complementizer *than* has a clausal complement composed of two parts each of which can be varied in in the way leading to a superlative construction. The best we can get is a comparative-anaphoric form of a superlative. When we vary the nominal argument (as in (20a) above) we get (30a) and when we vary the argument corresponding to the common noun we get (30b); by varying both arguments we get the CA superlative form in (31):

(30a) Bill bought more cars then anybody else (bought) books
(30b) Bill bought more cars that Leo anything else
(31) Bill bought more cars than anybody anything else.

When one looks at example (29) one realizes that even the comparative-anaphoric form cannot be obtained with the pattern it represents. At least not in a systematic way allowing easy linguistic reproductivity. Of course, as indicated above, the corresponding (higher order) function can easily be given in a more formal language. The problem why such functions are not (easily) denotable by superlatives in NLs is another topic. Maybe there are just no such things as negative superlatives.

Before analysing "ordinary" non-numerical comparatives I will briefly indicate two applications of the above proposal concerning the derivation of numerical superlatives from comparatives. I will discuss here only the question of why NLs, strictly speaking, do not have the negative form of $MOST$ with the proportional meaning, and, why some comparatives (especially non-numerical) are ambiguous in the way which has been indicated in many works on superlatives (the case of *the highest mountain*).

It has been often observed that $MOST$ does not have in its proportional reading a negative counterpart. In other words NLs do not have a determiner, or to be more precise a negative superlative, which would mean $LESS\text{-}THAN\text{-}HALF$. Of course languages have such a determiner: precisely *less than half* means this. Similarly *most* with the post (inner) negation (that is *most...not*) means this as well. What I am going to show is that there is no (syntactic) negative superlative with the above meaning.

The problem is the following: in English *the fewest* (intuitively the morphological superlative of *few*) has only relative meaning: (32) means (with the appropriate intonation) (33a) and not (33b) :

(32) Leo met the fewest Albanians.
(33a) Leo met fewer Albanians than everybody else.
(33b) Leo met less that half of Albanians.

The lack of such a determiner has given rise to more or less important discussions concerning learnability and the processing of proportional determiners, their formal properties (conservativity), etc. (cf. Hackl 2009, Pietroski *et al.* 2009). The proposal made here offers another solution to this problem.

Notice first that under the lacking reading $THE\text{-}FEWEST$ would be a "negative" superlative derived from the comparative based on type $\langle 1\langle 1,1\rangle\rangle$ quantifier $LESS$ (or its accusative extension when used in object position. Thus (34)- with proportional meaning, if it had one, would be derived from (35a) via the CA form given in (35b) and which is supposed to be equivalent to (35c):

(34) The fewest teachers danced.
(35a) Fewer teachers danced than sang.
(35b) Fewer teachers danced than did anything else.
(35c) Fewer teachers danced than sang and than smoke and than slept, etc

More formally, the meaning of such a superlative is given in (36):

(36) $THE\text{-}FEWEST(X)(Y) = 1$ iff $\forall Z(X \cap Z = \emptyset) \rightarrow |X \cap Y| < |Z \cap Y|$

Observe now that given this semantics, (36) can have only a trivial meaning in that it is always false since, we include among the possible values of Z the empty set. Even if we exclude this possibility and change (36) into (37) the situation is not much better:

(37) $THE\text{-}FEWEST(X)(Y) = 1$ iff $\forall Z(X \cap Z = \emptyset) \wedge (Y \neq \emptyset) \rightarrow |X \cap Y| < |Z \cap Y|$

In this case (37) is true only when X is empty. Thus (34) would be true only if there were no teachers. This is obviously far from what is desired. Thus the triviality of the meaning of the hypothetic superlative excludes its existence in NLs.

The difference between the positive proportional superlative discussed above and the negative just discussed is the equivalence in (9): there is no such equivalence for the negative superlative since the expression in (38) is false:

(38) For all sets X and Y, $|X \cap Y| > |X' \cap Y|$ iff $\forall Z(X \cap Z = \emptyset) \rightarrow |X \cap Y| < |Z \cap Y|$

Observe that we can use quite similar reasoning to explain the absence in NLs of the negative comparative with proportional reading related to the non-conservative superlative $MOSTLY$. In other words one can predict that no language has a superlative (a type $\langle 1,1\rangle$ determiner) $FEWESTLY$ with the meaning given in (38):

(39) $FEWESTLY(X)(Y) = 1$ iff less than half of Y are X

The reason is not that $FEWESTLY$ is not conservative ($MOSTLY$ is not conservative) but the fact that it woud have a trivial meaning in the same way as the superlative $THE\text{-}FEWEST$ on the proportional reading.

My proposal also explains why the existing negative superlatives with the relative reading cannot easily occur in subject position. This is the case with *the fewest* or with the German *die wenigsten*. We have seen that the positive (nominative) superlative with this meaning is derived not from a determiner froming an ordinary NP but from a type $\langle 2:1 \rangle$ function which is not an accusative extension of any type $\langle 1 \rangle$ quantifier. The same is true for the negative superlative with relative meaning. Such a function is involved in the semantics of the sentence in (40) which contains the comparative-anaphoric form of the superlative. The explicit value of this (type $\langle 2:1 \rangle$, since we suppose STUDENTS to be fixed) function is given in (41):

(40) Leo knows fewer students that anybody else.
(41) $F_A(R) = \{x : \forall(y \neq x)|xR \cap A| < |yR \cap A|\}$

For no non trivial A is this function an accusative extension (even though it satisfies a weaker condition given in (19). Consequently it is not denoted by an ordinary NP and hence the expression that denotes it does not behave like an ordinary referentially independent NP .

3 Non-numerical superlatives

Non-numerical comparatives and superlatives have many properties similar to those of numerical ones. One observes in particular that some superlatives can be derived from two distinct comparatives. There are, however, as in the case of numerical comparatives, some "multiple" comparative constructions which which do not give rise to superlatives. I will only discuss briefly ambiguous superlatives and show that they can be derived from different (non-numerical) comparatives which consequently induce different meanings causing the ambiguity of superlatives.

It has been observed (the classical paper concerning this problem is Szabolcsi 1986) that non-numerical comparatives are ambiguous, displaying either relative or absolute readings. For instance (42a) and (43a) are ambigous with the two meanings given respectively in (b) and (c):

(42a) Leo climbed the highest mountain.
(42b)Leo climbed the mountain which is higher than any other mountain.
(42c) Leo climbed the mountain which is higher than any other mountain climbed by anybody else
(43a) Leo hugged the oldest woman.
(43b) Leo hugged a woman who is older than any other woman.
(43c) Leo hugged a woman which is older than any other woman hug by anybody else.

The readings in the (b) sentences will be called absolute readings and those in (c), relative ones. It is easy to see that relative readings in particular are subject to various licensing conditions, a licenser being typically a focused phrase or a wh-phrase. In addition various other means proper to the informational structure of sentences may be used to distinguish the two readings more clearly (for some discussion see Szabolcsi 1986 and Hackl 2009). Furthermore, as usual when quantification and comparison is involved, additional limitations of the set of compared objects to those contextually relevant should be specified.

Ordinary superlatives can get additional readings in intensional contexts or with particular transitive verbs. Thus with verbs of creation we get an additional (kind of) *de dicto* reading as shown in the following example:

(44) Leo drew the highest mountain.

In this sentence one can compare the height of pictured or drawn mountains on the pictures and not in reality. We will ignore such intensional readings.

It is clear that different readings of superlatives in (42) and (43) are related to different comparatives from which they originate. Thus an explanation of these different readings is easy to conceive along the lines here proposed or similar ones (cf. Hackl 2009). However, more can be said about the formal status of the corresponding denotational functions. For this purpose it is important to notice that when the superlative occurs in subject position it can have only the absolute reading:

(45) The oldest woman lives in Japan.

The relative reading entails the existence of two participants in the action expressed by the transitive verb (since we "compare" participants in the action) and for that reason (45) has only absolute reading.

The above observation suggests that superlatives with relative readings are not related to an accusative extension of (the denotation of) any noun phrase, and thus in particular the superlative itself cannot be considered as an "ordinary" nous phrase denoting a type $\langle 1 \rangle$ quantifier. This is indeed the case since it does not satisfy the AEC condition given in (17). We show this informally, just using English examples, by showing that the corresponding CA form of superlative, the one given in (43c) does not satisfy AEC. Suppose that (46) holds:

(46) Persons that Leo hugged are the same as those that Bill kissed.

In (46) we have an instance of the conditional part of the AEC condition. One observes now that, given (46), the sentence in (47a) needs not hold the same truth value as the one in (47b):

(47a) Leo hugged a woman who is older than any other woman hugged by anybody else
(47b) Bill kissed a woman who is older than any other woman kissed by anybody else.

One can prove the above result more formally. For this we will use the necessary condition given in (48) for the AEC to be satisfied and consider just one conjunct, the one in (49), of the corresponding multiple conjunction. In the semantics of (49) the function $F_{O,a,W}$ of type $\langle 2:1\rangle$, defined in (50), is involved:

(48) If a type $\langle 2:1\rangle$ function F satisfies AEC then for any $A \subset E$, $F(E \times A) = E$ or $F(E \times A) = \emptyset$

(49) Leo hugged a woman older than any woman hugged by Bill.

(50) $F_{O,a.W}(R) = \{x : \exists y(y \in xR \cap W \wedge \forall z(z \neq y \wedge z \in aR \cap W) \rightarrow yOz\}$

In (50) W is the set of women, $a \in E$ and O corresponds to the relation *older than.*

Suppose now that $R = E \times W$. Then $F_{O,a.W}(R)$ is different from E and from $\emptyset$ which means that $F_{O,a.W}$ does not satisfy AEC.

On the other hand the function interpreting the absolute reading of the superlative does satisfy the AEC condition : from (46) follows the identity of truth values between (51a) and (51b) :

(51a) Leo hugged a woman who is older than any other woman.

(51b) Bill kissed a woman who is older than any other woman.

Finally, one observes that the function involved in the semantics of superlatives under the relative reading (for instance the one allowing the interpretation of (51b)) satifies the weaker condition given in (19).

4 Conclusive remarks

At first glance superlative phrases, whether numeric or non-numeric, look syntactically like noun phrases. Semantically, however, they may be ambiguous and the ambiguity occurs only in some syntactic environemets. In particular the respective readings of superlatives are correlated with the comparative constructions to which they are naturally related because superlatives are just multiple conjunctions of comparatives. As it happens, on the surface, comparatives can be related to, or can originate from, two different comparatives. One of these comparatives cannot occur in the subject position of a sentence. Similarly the related superlative is not ambiguous when occurring in subject position. Semantically, such comparatives and superlatives cannot be interpreted by functions corresponding to generalised (type $\langle 1\rangle$) quantifiers. Formally this amounts to saying that they do not satisfy the specific invariant condition for type $\langle 1\rangle$ quantifiers given in (17) above . They satisfy, however the strictly weaker condition given in (19). Consequently one can say that superlatives (and some comparatives) essentially augment the expressive power of, say, English, since the expressive power of English would be less that it is if the only noun phrases we need were ones interpretable as subjects of main clause intransitive verbs. The reason is that superlatives like *the oldest lady* (considered as the second nominal argument of transitive verbs) must be interpreted in their relative readings by functions from relations to sets which lie outside the class of generalised quantifiers as classically defined, that is type $\langle 2:1\rangle$ functions which are not extensions of type $\langle 1\rangle$ quantifiers.

One can effectively compute the size of the set of type $\langle 2:1\rangle$ functions satisfying the AEC condition and of the set of type $\langle 2:1\rangle$ functions which satisfy the argument invariance condition given in (19).

Observe first that both these sets form atomic Boolean algebras. Atoms of the algebra of functions satisfying the AEC are defined in (52):

(52) For any set A the function $F_A(R) = \{x : xR = A\}$ is an atom of the algebra of functions satisfying the AEC condition. Moreover, all atoms of this algebra have this form.

When $|E| = n$, there is 2^n of such atomic functions. Hence there is 2^m, $m = 2^n$, of functions satisfying the AEC.

Atoms of the algebra of functions satisfying the argument invariance condition are defined in (53):

(53) For any set A and any $a \in E$, the function $F_{a,A}(R) = \{x : xR = aR \cap A\}$ is an atom of the algebra of functions satisfying the argument invariance condition. Moreover, all atoms of this algebra have this form.

It follows from (53) that when $|E| = n$ there are $n \cdot 2^n$ atoms in the considered algebra. Hence there are 2^k, $k = n \cdot 2^n$ of all functions which are argument invariant.

It follows from the above results that in the world with just two individuals there are 16 possible extensions for referentially independent NPs whereas there are 256 of argument invariant functions.

It might be interesting to notice the analogy with nominal anaphors. Keenan (1987a, 1988, 2007) shows that something similar is true because of the existence of nominal anaphors. More specifically, anaphors like *himself, herself* (considered as the second nominal argument of transitive verbs) also must be interpreted by functions which do not satisfy the AEC and thus the generalised type $\langle 1\rangle$ quantifiers are not enough for their interpretation. Anaphoric functions interpreting anaphors satisfy another weakening of the AEC, the so-called predicate invariance (cf. Keenan and Westerståhl 1997) given in (54):

(54) A type $\langle 2:1\rangle$ function F is predicate invariant iff for any binary relation R and S, if $aR = aS$ then $a \in F(R)$ iff $a \in F(S)$

As we have seen, sets of functions satisfying (exactly) one of the three invariance conditions form Boolean algebras. The set of functions satisfying the AEC condition forms a (proper) sub-algebra of the algebra satisfying the predicate invariance as well as (proper) sub-algebra of the algebra of functions satisfying the predicate invariance. Thus the size of functions which can interpret superlatives and ordinary NPs is essentially greater than the size of the algebra corresponding to type $\langle 1\rangle$ quantifiers.

Obviously, the above results are meant to be language independent. The fact that various languages are "partially" sensitive at the syntactical level to the ambiguity of superlatives (Aihara 2009, Hackl 2009) cannot be considered as contradicting the results presented here.

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