Looking for origins of intensionality

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Abstract. Two interrelated problems are discussed. First, it is shown that the change in truth-value resulting from co-extensional substitution in sentences with intensional operators is due, roughly speaking, either to the violation of some lexical presuppositions proper to intensional operators or to the violation of some "natural" generic truths. Secondly, an algebraic description of such changes in truth-value is given.

1 Introduction

Research in natural language semantics shows that resistance to substitution (by co-extensional expressions), and more generally intensionality, is a very frequent phenomenon (Partee 1974, Keenan and Faltz 1985, Saul 1997, Larson 2002, Zuber 2006). What linguists have discovered, however, is that non-substutivity can occur in many other contexts than those related to attitude ascriptions and, more generally, in contexts of specific expressions which do not have any cognitive or even psychological content. Furthermore, it has been noted that non substituable expressions need not be definite descriptions (or nominals in general). This paper has a twofold purpose. First I will show that, contrary to what is generally assumed, the substitution of expressions by co-extensional ones in intensional contexts is not innocuous and often leads to subtle changes in meaning. Thus I will show that substitution failure has a specific semantic ground and is due essentially to the fact that a substitution may lead to the negation of a (semantic) presupposition which ia a part of the lexical meaning of the intensional expression. It may also be due to the denial, provoked by the substitution, of a specific generic truth. Second, I will try to give a general (algebraic) characterisation of intensional functional expressions, that is functional expressions whose arguments cannot be freely (i.e. without changing the semantic value of the whole) substituted by co-extensional expressions. It will follow from this characterisation that intensional functional expressions form a sub-class of syncategorematic expressions.

Obviously, given the variety of intensional constructions many of them will not be taken into consideration in what follows. Although my proposal also applies to "classical" cases of intensionality of verbs of propositional attitude (that is, syntactically, verbs taking sentences as one of their arguments) I will be basically concerned with specific intensional transitive verbs (essentially verbs of absence), intensional adjectives and adverbs, and with probably less known cases of intensionality induced by some intensional prepositional phrases and some

scalar particles. I will not discuss intensional operators which are quotational in nature.

One final general remark. In this paper I am interested in constructions, essentially functional expressions, which are resistant to the semantically harmless substitutivity of their arguments. It is customary to consider additionally as a criterion for intensionality, especially when noun phrases are involved, the failure of existentialisation or the possibility of the existential reading of indefinites. It is well-known by now that the two criteria do not make the same discrimination and fail to distinguish adequately (Moltmann 1997): one expression can be considered as intensional according to one criterion but not according to the other. Consider, just for illustration, the verb *to try* when it takes an infinitival (with a direct object) complement: the expressions *Leo tried to solve the problem* P_1 and *Leo tried to solve the problem* P_2 may have different truth values even if P_1 and P_2 is logically the same existing problem. The reason is that the person trying to solve the problem P_1 may not be aware of the fact that this problem is logically equivalent to the problem P_2 and his intention was to solve precisely the problem P_1. Thus *to try* is probably intensional according to the substitution criterion but not intensional according to the existence criterion (see Sharvit 2003 for the discussion of intensionality of *try* according to the second criterion). In what follows I will be interested only in the failure of substitutivity in intensional contexts and I will not try to establish any relationship between the two criteria.

2 Formal preliminaries

The theoretical tools which will be used in my analysis are those of Boolean semantics (Keenan and Faltz 1985): with every grammatical category C is associated its denotational denotational algebra (d-algebra) D_C, which is a set of possible denotations of expressions of category C. D-algebras D_C form atomic Boolean algebras. Elements of $D_{B/A}$ are functions from D_A to D_B and Boolean operations on them are defined pointwise. There are various restrictions on such functions. These restrictions give rise to various sub-algebras (which correspond to denotations of sub-categorised expressions). The fact that possible denotations for expressions of a given (not necessarily sentential) category have a Boolean structure allows us to talk about cross-categorial entailment, that is an entailment between expression of that (not necessarily sentential) category. This generalised entailment corresponds to the Boolean order proper for the given d-algebra.

Given a Boolean algebra B and an $a \in B$ it is possible to form an algebra $B(a)$, called the algebra restricted by a, in the following way: $B(a) = \{x : x \leq a\}$, the meet, the join and the zero in $B(a)$ are the same as in B and the unit element in $B(a)$ equals to a. Consequently the complement $c(x)$ of an element x in $B(a)$ must be relativised to a. In other words for the complement relativised in this way the following holds: $c(x) = x' \cap a$, where x' is the Boolean complement of x.

As an example of restricted algebras consider the algebra $D_{CN}(H)$ which is the algebra of possible denotations of common nouns restricted by the property H (the denotation of *human being).* Thus $D_{CN}(H)$ corresponds to the set of possible denotations of CNs all of which entail *human being* (there are for instance CNs corresponding to "professional activities"): if *teacher* denotes T then *non-teacher* denotes $c(T)$ which is the set of all objects which are not teachers but are humans.

It can be seen that the restricting element corresponds to the (lexical) presupposition of expressions which denote in a given restricted algebra. For instance *teacher, student, doctor*, etc. presuppose *human being* because these items and their (relativised) negation each cross-categorially entail *"human being.*

We will make use of restricted denotational algebras in which expressions of other categories than common nouns denote. All such algebras are atomic. Since we will consider intensional adjectives we will be interested in the denotational algebras of modifiers. A modifier is a functional expression of category C/C for various choices of C. Thus, given the algebra $D_{C/C}$ of all functions from D_C onto D_C we can consider a restricted algebra of it in which the restricting element β equals the identity function id_c. For such algebras we have:

Proposition 1: Let B be a Boolean algebra. The set of functions f from B onto B satisfying the condition $f(x) \leq x$ forms a Boolean algebra R_B with the Boolean operations of meet and join defined pointwise: $0_{R_B} = 0_B$, $1_{R_B} = id_B$, $f'(x) = x \cap (f(x))'$. If B is atomic so is R_B. For all $b \in B$ and all atoms α of B such that $\alpha \leq b$, functions $f_{b,\alpha}$ defined by $f_{b,\alpha}(x) = \alpha$ if $x = b$ and $f_{b,\alpha}(x) = O_B$ if $x \neq b$ are the atoms

The restricted algebra R_B has a (non-restricted) sub-algebra $ABS(B)$ of *absolute functions.* By definition $f \in ABS(B)$ iff for any $x \in B$, we have $f(x) = x \cap f(1_B)$. For absolute functions the following holds:

Proposition 2: $ABS(B)$ is a sub-algebra of R_B. If B is atomic so is $ABS(B)$. For all atoms α of B functions f_α defined by $f_\alpha(x) = \alpha$ if $\alpha \subseteq x$ and $f_\alpha(x) = O_B$ otherwise, are the atoms of $ABS(B)$

The difference between absolute and non-absolute restricted algebras can be illustrated by the following example: if we consider adjectives (expressions of category CN/CN) then absolute functions are denotations of absolute (non-gradable or intersective) adjectives (which are not intensional) and non-absolute restrictive functions are denoted by non-gradable (non-intersective) adjectives (which are often intensional). Thus *red match* cross-categorially entails *match* and *red object* whereas *big match* cross-categorially entails *match* but does not entail *big object.*

Propositions 1 and 2 show that both types of modifier algebras are atomic. Concerning atoms defined above we observe that atoms denoted by modifiers are always determined by, or are "indexed" by, some elements (usually the atoms)

of the d-algebra of the modified expressions. In the case of restricted algebras atomic functions are even double indexed. Thus in particular in general non-trivial values of atomic functions are determined by their index which may depend on the possible argument. This means that functional expressions denoting atoms are *syncategorematic* in the sense that their full description depends (for their index) on the value of the argument to which they apply (Zuber 2004a). For instance one can show that the categorially polyvalent modifier *only* denotes an atom in the corresponding absolute d-algebra: if *only* modifies NPs then it denotes an atomic function in the d-algebra $D_{NP/NP}$ of absolute functions. Which exact function is denoted depends on the argument NP which gives the index for the function. The full expression *Only NP* denotes an atom in D_{NP}, which is a set containing containing just one set as element (this set is belongs to the quantifier denoted by NP).

From the empirical and theoretical points of view, the observation about natural languages is that atomic modifiers (i.e. modifiers denoting atoms) are syncategorematic. Consider for instance the NP *only Leo*. It denotes the atomic quantifier (of type $< 1 >$), member of D_{NP}. It contains a unique property $\{L_1\}$, which is a singleton containing the referent of *Leo*, as its only element. Similarly, *only Leo and Lea* denotes the atomic quantifier whose only member is the two element set $\{L_1, L_2\}$. This means that *only* in *only Leo* denotes an atomic function which is different from the one denoted by *only* in *only Leo and Lea*. Indeed, the first function is determined by *Leo* and is thus indexed by the singleton $\{L_1\}$, whereas the second function is determined by *Leo and Lea* and thus indexed by the two element set $\{L_1, L_2\}$. So *only* is a syncategorematic expression. Indeed one of the meaning components of *only x* is expressed by the clause "Nothing different from x" which is clearly syncategorematic. Notice, however, that *only*, which denotes an absolute function, is not intensional (at least when it applies to proper names). Indeed, the denotation of *only Leo* will not change if we replace *Leo* by a co-referential NP. We will see that the situation is not the same with non-absolute atomic modifiers since they are usually denoted by intensional functional expressions.

The last notion which will be used in my analysis is the notion of *generic incompatibility* or *generic compatibility*. Given our poor knowledge of the semantics of generic sentences this distinction will not be made very precise nor justified. Thus in the same way as with logical compatibility, incompatibility or inclusion there is a *generic* incompatibility, compatibility or inclusion. More precisely I assume that there is a class of properties such that with each of its members one can associate a property generically compatible (g-compatible) with it (or generically incompatible with it). More specifically a property P_2 is g-compatible with the property P_1 iff the generic sentence corresponding to P_1 *are* P_2 (or some version of this form) is (generally considered as) true. For instance *to be a dancer* is g-compatible with *be two-legged* because the sentence *Dancers are two-legged* is generically true. On the other hand *be one-legged* is g-incompatible with *be a dancer*. G-compatibility and g-incompatibility are of course in some sense intensional notions (given that genericity involves intensionality).

3 Analysis

I will now analyse various cases of intensional expressions with the help of the notions introduced in the previous section in order to show that substitution in intensional contexts may lead to some semantic side effects. More specifically this analysis will show that the substitution of an expression in an intensional context by a co-extensional expression leads either to a violation of presupposition or to the denial of some "natural" generic truth.

Consider first the class of transitive intensional verbs (TIV). Although members of this class are often used to illustrate specific intensional constructions it does not mean this is a homogeneous class or that there is general agreement concerning its membership or even its existence (cf. Moltmann 1997, Saul 2002, Forbes 2004). In my proposal I will be concerned with a proper sub-class of all TIV, the sub-class whose members have a specific syntactic property.

Traditionally TIV includes verbs like *want, wish, hope, fear, despise, like*, etc. Sometimes, additionally one includes in this class also the verb like *to own* and *to worship*. All these verbs are, roughly speaking, transitive. One observes, however, that there is a categorial difference between the two classes of verbs. Thus only the verbs of the first class are *categorially polyvalent*, that is only the verbs of the first class can take arguments (in the object position) of various grammatical categories. For instance the verb *fear* can take an NP (*fear the professor*), an infinitival clause (*fear to swim*), and a *that*-clause (*fear that S*) as complement. Similarly with other members of the first class. This is not the case with *to own* and, to a lesser degree, with *to worship* (for the discussion of some specific properties of these last two verbs see Moltmann 1997 and Zimmermann 2002). I will take into account only the categorially polyvalent TIVs which exhibit the failure of substitution independently of the grammatical category of their complement.

The proposal I want to make is based on the following observation: all categorially polyvalent TIVs (and usually their negation) imply, generally speaking, a specific knowledge by the person referred to by their possible grammatical subject. This knowledge concerns the object denoted by an NP in their complement or the action expressed by the complement. For instance (1a) presupposes (1b) and (2a) probably presupposes (2b):

(1a) Leo fears the professor.
(1b) Leo knows (some properties of) the professor.
(2a) Leo fears to swim.
(2b) Leo knows how to swim (what swimming is).

Thus *to fear, to look for, to like, to need something* entails (on the appropriate reading) knowing what that something is (or to know that "something" or the corresponding action). This knowledge constitutes the restricting element of the denotational algebra of TIV. Of course this element is not a linguistic (syntactic) object but a semantic object, a member of a d-algebra. Given the fact that TIVs considered are categorially polyvalent there will be many, pos-

sibly functionally related, such restricting elements, each corresponding to the possible grammatical category that a given TIV can have given the category of the argument it takes.

It is easy to check that when the substitution of an NP (or a constituent of another category) in the complement of TIV fails then precisely the presupposed knowledge is violated. Thus if *A likes/fears* NP_1 differs in truth value from *A likes/fears* NP_2 (and NP_1 and NP_2 have the same extension) then the knowledge A has about NP_1 is not the same that A has about NP_2.

Notice incidentally that "classical" verbs of propositional attitude have a similar property. Thus in general one observes that the reason that sentences with the verb *to believe*, when they take a sentential argument, change their truth value from true to false is precisely that the grammatical subject of such sentences (which refers to human beings) lack specific knowledge about an object to which the replaced constituent or some of its parts may refer.

One might object that the above observation concerning knowledge does not hold for all cases since possibly one can fear "something unknown" or one can fear to do something without knowing how to do this. My reply is that such situations are in some sense exceptional and that "as a rule" or "generically" the indicated implications between (1a) and (1b) or between (2a) and (2b) hold. This observation leads us to the second possible origin of intensionality, which is the violation of some "obvious" generic truths to which substitution may give rise.

To illustrate how violation of generic truths is related to non-substitutivity we will consider various modifiers. Consider first an atomic non absolute modifier. As we have seen, such modifiers denote atoms which are doubly indexed. One can show that *even* is such a modifier (Zuber 2004b). Indeed *Even Leo danced* entails that Leo danced and additionally conveys the meaning (the *surprise effect*) that this fact is surprising. This surprise effect can be expressed "algebraically" in the following way (cf. Zuber 2004b): there is a property not g-compatible (or maybe g-incompatible) with dancing and such that among all dancers only Leo has it. So *even* in this example denotes an atom which is indexed by *Leo* (its denotation) and a property which is not g-compatible with dancing (the property of the VP phrase).

An interesting fact is that *even* induces intensionality (Zuber 2006):

(3a) Even Leo danced.
(3b) Even Leo sang.

The above sentences need not have the same truth value even if the set of singers and the set of dancers is the same (if one considers that the surprise effect induced by *even* corresponds to a semantic content). This is because properties g-compatible with dancing are not necessarily g-compatible with singing: in (3a) Leo is the only dancer having the property P_1, g-incompatible with dancing and in (3b) Leo is the only singer having the property P_2 g-incompatible with singing. Thus the atom denoted by *even* in (3a) is determined by the denotation of Leo

and by ther property P_1 and the atom denoted by *even* in (3b) is determined by the denotation of Leo and the property P_2.

We can apply the same move to other so-called intensional modifiers. It is easy to realise that (4a) needs not to have the same truth values as (4b) in the same way as (5a) needs not to have the same extension as (5b) even if the set of dancers and the set of boxers is the same:

(4a) Five skilful dancers were at the party.
(4b) Five skilful boxers were at the party.
(5a) Beautiful dancer
(5b) Beautiful boxer

The adjective *beautiful* being non-absolute we understand why *beautiful dancer* and *beautiful boxer* do not have the same extension (even if the set of dancers and the set of boxers is the same). The reason is that the set of generic truths about dancers is not the same as the set of generic truths about boxers.

Various intensional adverbs (*skilfully, slowly*, etc.) can be analysed in exactly the same way because adverbs are VP modifiers, and intensional modifiers denote non-absolute restrictive functions.

Finally consider the less well known case of intensionality of some prepositional phrases. It has been noticed (Keenan and Faltz 1985) that comitative, directional, instrumental, subject-matter (with the preposition *about* PPs are not transparent. For instance predicates *working with Leo* and *talking with Leo* need not to have the same extension in the situation in which *working* and *talking* do have the same extension. Thus the comitative phrase *with Leo* induces intensionality. Similarly subject-matter PPs (NPs preceded by *about*) are intensional: those who are thinking about intensionality need not be the same as those who are talking about intensionality even if the individuals who are talking are the same as the individuals who are thinking. Similar examples can be given for many other types of PPs (see Keenan and Faltz 1985). What is important, however, is the fact that not all PPs induce intensionality. For instance locative PPs are transparent. Thus *dance in the garden* and *sing in the garden* have the same extension if *dance* and *sing* have the same extension (the set of dancers is the same as the set of singers). And indeed it is not easy to find a property which would be g-compatible with *dance in the garden* and not with *sing in the garden*. Or, as observed in Keenan and Faltz (1985), locative PPs are modifiers which denote absolute functions (*to sing in the garden* is *to sing and be in the garden*) whereas other PPs, those inducing intensionality, are modifiers which denote restrictive non absolute functions.

4 Conclusions

I have tried to show what are the origins, at least in some cases, of non-substitutivity. From the empirical point of view it has been shown that the falsity one gets in cases where substitution by co-extensional expressions fails, is

not "ordinary" falsity but falsity due to presupposition violation or to the negation of generic truths (or both). This means that in the context of substitution failure there is no "ordinary" symmetry between the truth and the falsity. The supposed falsity after the substitution is not due to the difference in the situation in the world but rather to some metalinguistic reasons. In another words it is not the assertion of the sentence which changes its truth value after substitution. This is related to the asymmetric (with respect to negation) status of sentences with a false presupposition and corresponds to the fact that in general the change of the truth value provoked by substitution goes from truth to falsity and not from falsity to truth.

I propose to explain these facts more formally by making use of restrictive Boolean algebras as d-algebras for intensional expressions. In these algebras the complement, and thus indirectly the negation, is not defined pointwise but is relativised by the restricting element. This relativising element, which is the maximal element of the restricted algebra (its "unit" element) usually corresponds to a natural feature that many linguistic elements may have. In the case of intensional constructions this feature corresponds to specific linguistic knowledge, which should be considered as presupposed. The substitution may lead to the violation of this knowledge.

I have also suggested that functional expressions denoting atoms of restrictive non-absolute algebras are intensional expressions (in the sense that substitutivity on their argument position fails), which are different from those presupposing knowledge. The reason is that such atoms are doubly indexed and one index is determined by a specific property (related to genericity or to presupposition). This relation with atomicity seems to be a sufficient but not a necessary condition for the intensionality to arise: adverbs and PPs inducing intensionality apparently need not denote atoms. Although I do not say much about generic sentences I relate implicitly sources of intensionality (non-substitutivity) to sources of genericity and of presuppositions. Finally use of atomic functions interpreting syncategorematic expressions in NLs allows us to treat intensionality as a special case of syncategorematicity. Indeed, intensionality of functional expressions is a special case of the dependence of their meaning on the meaning (technically "intension") of their possible arguments.

Notice that the explanation offered here for substitution failure is of a semantic nature: it involves a semantic presupposition and a generic truth. It thus differs from some pragmatic explanations which claim that different conversational implicatures should be attached to sentences with propositional attitude verbs differing just by the presence of different but co-extensional component expressions (see Spencer 2006). I do not claim, however that all cases of intensionality can be explained without the use of pragmatics. This seems in particular to be the case with the non-substitutivity of proper names by other, supposedly co-referential proper names in some very marked pragmatically contexts (Saul 1997, Zimmermann 2005).

On the other hand it seems to me that other cases of intensionality can be analysed along the lines proposed here. Consider for instance a NP modifier like

as as it occurs in *to work as a CN*. If we consider the common nouns in such constructions as arguments then clearly the remaining part is an intensional expression: obviously *to work as a teacher* and *to work as a lawyer* may differ in extension even if the set of lawyers and teachers is the same. At the same time, we observe that generic truths one can associate with various predicates obtained by the application of *as* to a NP vary.

Of course whatever the examples to which one would like to apply my proposal, many details need to be spelled out and its exact implementation needs to be made precise.

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