Remarks on Kratzer's *Ordering-Premise* Semantics of Modalities and Conditionals

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Definitions of Ordering-Premise Semantics

Let W be a set of possible worlds, functions such that

$\mathcal{B}, \mathcal{O} : W \to \wp(\wp(W))$

are called a *modal base* and *ordering source* respectively.
They gives two types of premise set (set of propositions) to each possible worlds.

Let $\Phi(\ni p)$ be a set of propositional variables,
then formula $\phi \in \mathcal{L}_{\Box,>,\Phi}$ is defined by:

$$\phi ::= p \mid \phi_1 \to \phi_2 \mid \phi_1 > \phi_2 \mid \Box\phi \mid \bot$$

where $\phi_1 \to \phi_2$ is a material implication which can define $\phi_1 \wedge \phi_2, \phi_1 \vee \phi_2, \neg\phi$ with $\bot$,
and $\phi_1 > \phi_2$ is a conditional formula.

Let $V : \Phi \to \wp(W)$ be a valuation of Φ.

Then $\mathcal{K} = \langle W, \mathcal{B}, \mathcal{O}, V \rangle$ is called

an Ordering-Premise Model of $\mathcal{L}_{\Box,>,\Phi}$

if $[\![\bullet]\!]_{\mathcal{K}} : \mathcal{L}_{\Box,>,\Phi} \to \wp(W)$ is defined by recursion on ϕ:

$[\![p]\!]_{\mathcal{K}} = V(p)$;

$[\![\phi_1 \to \phi_2]\!]_{\mathcal{K}} = (W \setminus [\![\phi_1]\!]_{\mathcal{K}}) \cup [\![\phi_2]\!]_{\mathcal{K}}$;

$[\![\bot]\!]_{\mathcal{K}} = \varnothing$;

$[\![\Box\phi]\!]_{\mathcal{K}} = \{ w \in W \mid$

$\forall u \in \bigcap \mathcal{B}(w) \exists v \in \bigcap \mathcal{B}(w).$

$v \leq_{\mathcal{O}(w)} u \;\&$

$\forall z \in \bigcap \mathcal{B}(w)\, z \leq_{\mathcal{O}(w)} v \Rightarrow z \in [\![\phi]\!]_{\mathcal{K}} \}$,

where

$u \leq_{\mathcal{O}(w)} v \Leftrightarrow \{ \boldsymbol{p} \in \mathcal{O}(w) \mid v \in \boldsymbol{p} \} \subseteq \{ \boldsymbol{p} \in \mathcal{O}(w) \mid u \in \boldsymbol{p} \}$.

Lemma 1.

(1) $\leq_{\mathcal{O}(w)}$ is reflexive, transitive, and anti-symmetry, even if $\mathcal{O}(w) = \varnothing$.

(2) $\leq_{\mathcal{O}(w)} \circ \leq_{\mathcal{O}(w)}$ is reflexive, transitive, and anti-symmetry, even if $\mathcal{O}(w) = \varnothing$.

(3)$w \leq_{\mathcal{O}(w)} w$ even if $\mathcal{O}(w) = \varnothing$.

Kripke Semantics

W : a non-empty set of possible worlds

R : a binary relation on W

$V : \Phi \rightarrow \wp(W)$ a valuation of Φ

$\mathcal{M} = \langle W, R, V \rangle$: a Kripke model

$\mathcal{M}, w \vDash \Box\phi \Leftrightarrow \forall u \in W . wRu \Rightarrow \mathcal{M}, u \vDash \phi$

$\mathcal{M}, w \vDash \Diamond\phi \Leftrightarrow \exists u \in W . wRu \,\&\, \mathcal{M}, u \vDash \phi$

$\Gamma \vDash \phi \Leftrightarrow: \forall \varphi \in \Gamma \forall w \in W . \mathcal{M}, w \vDash \varphi \Rightarrow \mathcal{M}, w \vDash \phi$

Lemma 2.

(1) $R_{\mathcal{K}} = \{(w,z) \mid$
$\forall u \in \bigcap \mathcal{B}(w) \exists v \in \bigcap \mathcal{B}(w) z \in \bigcap \mathcal{B}(w). z \leq_{\mathcal{O}(w)} v \,\&\, v \leq_{\mathcal{O}(w)} u\}$
$= \{(w,z) \mid \forall u \in \bigcap \mathcal{B}(w) z \leq_{\mathcal{O}(w)} \circ \leq_{\mathcal{O}(w)} u\}$
is reflexive, transitive, and serial if $w \in \bigcap \mathcal{B}(w)$ $\because$ Lemma 1(2)

(2) If $\forall w \in W : \mathcal{B}(w) = \varnothing$, then $\langle W, R_{\mathcal{K}} \rangle$ is a reflexive, transitive, serial Kripke frame
$\because R_{\mathcal{K}} = \{(w,z) \mid \forall u \in W. z \leq_{\mathcal{O}(w)} \circ \leq_{\mathcal{O}(w)} u\}$.

(3) $R_{\mathcal{K}}$ is transitive and serial $\because$ Lemma 2(1-2).

Lemma 3.

(1) $\langle W, R_{\mathcal{K}} \rangle \vDash \Box\varphi \to \Diamond\varphi$ $\because$ Lemma 2(3),

(2) $\langle W, R_{\mathcal{K}} \rangle \vDash \Box\varphi \to \Box\Box\varphi$ $\because$ Lemma 2(3),

(3) $\langle W, R_{\mathcal{K}} \rangle \vDash ?\mathcal{X} \to (\Box\varphi \to \varphi)$ $\because$ Lemma 2(1-2)

Problems on Ordering-Premise Semantics

- Case1:Some verifiable propositions are not in any premise sets
- Case2:On "Overcoming Inconsistencies"
- Case3:On "Practical Inference"=multi-modal inference

Case 1:Circumstantial Conversational Backgrounds

$\mathcal{B}$ is realistic, i.e., $\forall w.w \in \bigcap \mathcal{B}(w)$.

$\mathcal{B}(u) = \{\mathbf{p}, \mathbf{q}\}$

$\mathcal{O}(u) = \{\mathbf{p}'''\}$,

$\mathbf{p} \cap \mathbf{q} = \{u\}, u \in \mathbf{p}' \subset \mathbf{p}''' \subset \mathbf{p}''$

Then, $\langle \mathrm{W}, \mathcal{B}, \mathcal{O}, V \rangle, u \vDash \Diamond \mathbf{p}', \Diamond \mathbf{p}''$

even though $\mathbf{p}'$ and $\mathbf{p}''$ are not

in the conversational background.

Case 1:Circumstantial Conversational Backgrounds

$u \in \mathbf{p}''' \cap \mathbf{q}' \subset \mathbf{p}''' \subset \mathbf{p}''' \cup \mathbf{q}'$

Then, $\langle W, \mathcal{B}, \mathcal{O}, V \rangle, u \vDash \Diamond(\mathbf{p}''' \vee \mathbf{q}'), \Diamond(\mathbf{p}''' \wedge \mathbf{q}')$

for each $\mathbf{q}'(\ni u)$ which is not

in the conversational background.

Case 2: Judgments

Judge A: a is liable (l); Judge B: a is not liable($\neg l$)

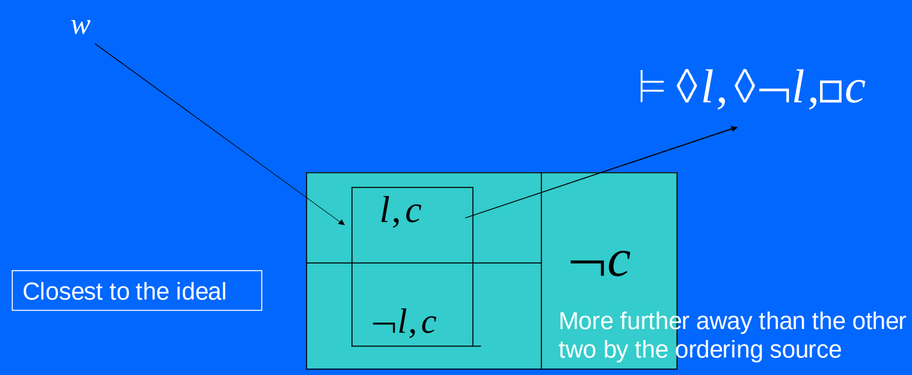
Judge A=Judge B: the murder is a crime (c)

$\therefore\ [\![\text{Judgments}]\!] = [\![\bot]\!] = \emptyset.$

Judgments $\vDash \Box c, \Box\neg c$

Judgments $\nvDash \Diamond l, \Diamond\neg l$, for all formula $c, l, \ldots$

Case 2: Judgments (Kratzer's Solution)



$\forall w.\mathcal{B}(w)=\emptyset$, i.e., $\bigcap \mathcal{B}(w)=W$, i.e., $R_{\mathcal{K}}$ is reflexive, i.e., w is impossible

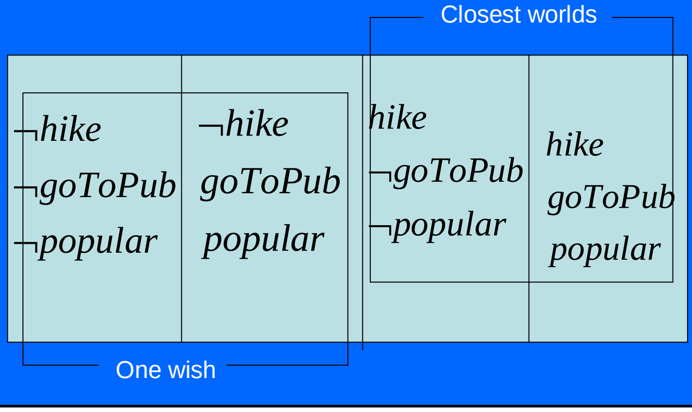
Case 3: Practical Inference

$\mathcal{B}$ is circumstantial, i.e., $\mathcal{B}(w)=\{goToPub \leftrightarrow popular\}$

$\mathcal{O}$ is bouletic, i.e., $\mathcal{O}(w)=\{popular, \neg goToPub, hike\}$

$\mathcal{B}(w)\cup\mathcal{O}(w)=[\![\bot]\!]=\emptyset$

Case 3: Practical Inference (Kratzer's Solution)



⊨ should *hike*,
could *go*,
could *pop*
could¬*go*,
could¬*pop*

Case 3: Practical Inference (Frank's Solution)

"Does this correspond to our intuitions? In one way it does, but in another it doesn't." Frank(1997)

She proposes a contraction of 'p' and 'not-p' in the sense of Belief Revision.

$\mathcal{B}(w) \cup !\mathcal{O}(w) = \mathcal{B}(w) \cup \mathcal{O}'(w),$

where

$\mathcal{O}'(w) \subseteq \mathcal{O}(w) \;\&\; \bigcap(\mathcal{B}(w) \cup \mathcal{O}'(w)) \neq \emptyset \;\&$

$\forall X \subseteq \mathcal{O}(w)$ if $\mathcal{O}'(w) \subseteq X$, then $\bigcap(\mathcal{B}(w) \cup X) = \emptyset.$

For another contraction in DRT, see 4.1.4. of Frank(1997).

Case 3

- If Kratzer's treatment of practical inference is right, we can add more multi-modal premises:

i. The speaker's father's wish: not-hike

ii. The law: not-gotoPub

iii. …

How many are ordering sources or modal bases needed? Or Such a multi-modal situation can be treated?

Lemma 4.

$$\mathcal{K}, w \vDash \Diamond\varphi$$
$$\Leftrightarrow \mathcal{K}, w \vDash \neg\Box\neg\varphi$$
$$\Leftrightarrow \neg\forall u \in \bigcap\mathcal{B}(w)\exists v \in \bigcap\mathcal{B}(w)\forall z \in \bigcap\mathcal{B}(w).$$
$$z \leq_{\mathcal{O}(w)} v \,\&\, v \leq_{\mathcal{O}(w)} u \Rightarrow \mathcal{K}, v \vDash \neg\varphi$$
$$\Leftrightarrow \exists u \in \bigcap\mathcal{B}(w)\forall v \in \bigcap\mathcal{B}(w)\exists z \in \bigcap\mathcal{B}(w).$$
$$z \leq_{\mathcal{O}(w)} v \,\&\, v \leq_{\mathcal{O}(w)} u \,\&\, \mathcal{K}, v \vDash \varphi$$

Ordering Lemma

$(1) v \leq_{\mathcal{O}(w)} u$ even if $\mathcal{O}(w) = \varnothing$

$\because P(u) \cap \mathcal{O}(w) = \varnothing \subseteq \varnothing = P(v) \cap \mathcal{O}(w)$

$(2) v \leq_{\mathcal{O}(w)} u$ even if $\mathcal{O}(w) \cap P(u) = \varnothing$

$\because P(u) \cap \mathcal{O}(w) = \varnothing \subseteq X$ for all X

$(3) v \leq_{\mathcal{O}(w)} u$ even if $\mathcal{O}(w) \cap P(u) = \varnothing \& P(v) \cap \mathcal{O}(w) = \varnothing$

$\because \varnothing \subseteq \varnothing$

Propositions on Case 2

$\mathcal{O}(w) = \{\neg l, l, \neg c\}$

$P(v) = \{\neg l, \neg c\} \therefore P(v)_{\mathcal{O}(w)} = \{\neg l, \neg c\}$

$P(u) = \{l, \neg c\} \therefore P(u)_{\mathcal{O}(w)} = \{l, \neg c\}$

$P(t) = \{\neg l, c\} \therefore P(t)_{\mathcal{O}(w)} = \{\neg l\}$

$P(s) = \{l, c\} \therefore P(s)_{\mathcal{O}(w)} = \{l\}$

$\mathcal{B}(w) = \varnothing \therefore \bigcap \mathcal{B}(w) = W (P(w) = \{\bot\}?)$

$\therefore u \leq_{\mathcal{O}(w)} s; v \leq_{\mathcal{O}(w)} t; x \leq_{\mathcal{O}(w)} x; (x \leq_{\mathcal{O}(w)} w)$

$\therefore u, v, t, s, w \leq_{\mathcal{O}(w)} t, s, w \leq_{\mathcal{O}(w)} w$

$\therefore w \vDash \Diamond l, \Diamond \neg l, ? \Diamond \neg c, ? \bot, ? \Diamond \bot$

Ordering Proposition: Case 3

$\mathcal{O}(w) = \{pop, \neg go, hike\}; \forall w.\mathcal{B}(w) = \{go \leftrightarrow pop\}$

$P(v) = \{\neg hike, \neg go, \neg pop\} \therefore P(v)_{\mathcal{O}(w)} = \{\neg go\}$

$P(u) = \{\neg hike, go, pop\} \therefore P(u)_{\mathcal{O}(w)} = \{pop\}$

$P(t) = \{hike, \neg go, \neg pop\} \therefore P(t)_{\mathcal{O}(w)} = \{hike, \neg go\}$

$P(s) = \{hike, go, pop\} \therefore P(s)_{\mathcal{O}(w)} = \{hike, pop\}$

Let $P(w) = \varnothing$ or $\{go \leftrightarrow pop\} \therefore P(w)_{\mathcal{O}(w)} = \varnothing$

Let $\bigcap \mathcal{B}(w) = \{w\}$ or W

$\therefore t \leq_{\mathcal{O}(w)} u, v; s \leq_{\mathcal{O}(w)} u; x \leq_{\mathcal{O}(w)} x; x \leq_{\mathcal{O}(w)} w$

$\therefore w \vDash \Diamond\neg go, \Diamond hike, \Diamond pop, \Diamond go, \Diamond\neg pop, \Box(go \leftrightarrow pop)$

Conditionals in Ordering-Premise Semantics

$\mathcal{K} = \langle W, \mathcal{B}, \mathcal{O}, V \rangle$

$[\![\phi_1 > \phi_2]\!]_{\mathcal{K}} = \{w \in W \mid w \in [\![\Box\phi_2]\!]_{\mathcal{K}[\mathcal{B}'/\mathcal{B}]}\}$,

where $\forall w \mathcal{B}'(w) = \mathcal{B}(w) \cup \{[\![\phi_1]\!]_{\mathcal{K}}\}$,

$= \{w \in W \mid \forall u \in \bigcap(\mathcal{B}(w) \cup \{[\![\phi_1]\!]_{\mathcal{K}}\}) \exists v \in \bigcap(\mathcal{B}(w) \cup \{[\![\phi_1]\!]_{\mathcal{K}}\})$

$\forall z \in \bigcap(\mathcal{B}(w) \cup \{[\![\phi_1]\!]_{\mathcal{K}}\}).z \leq_{\mathcal{O}(w)} v \,\&\, v \leq_{\mathcal{O}(w)} u \Rightarrow z \in [\![\phi_2]\!]_{\mathcal{K}}\}$

(1) material: $\forall w \bigcap \mathcal{B}(w) = \{w\}; \forall w \mathcal{O}(w) = \varnothing$.

(2) strict: $\forall w \mathcal{B}(w) = \varnothing; \forall w \mathcal{O}(w) = \varnothing$.

(3) counterfactual: $\forall w \mathcal{B}(w) = \varnothing; \forall w \bigcap \mathcal{O}(w) = \{w\}$.

(4) indicative: $\forall w \mathcal{B}(w) = \varnothing; \mathcal{O}$ is circumstantial

Counterfactuals in Ordering-Premise Semantics

- A counterfactual is characterized by an empty modal base *f* and a totally realistic ordering source *g*. It follows from David Lewis' work mentioned above, that this analysis of counterfactuals is equivalent to the one I give in Kratzer(1981b). …The idea is this: All possible worlds in which the antecedent *p* is true, are ordered with respect to their being more or less near to what is actually the case in the world under consideration. – Kratzer(1981a)

Standard Semantics of Conditionals (SMC)

$\mathcal{M} = \langle W, f, V \rangle;$

$f : W \times \wp(W) \rightarrow \wp(W)$ (selection function)

$[\![\phi_1 > \phi_2]\!]_{\mathcal{M}} = \{ w \in W \mid f(w, [\![\phi_1]\!]_{\mathcal{M}}) \subseteq [\![\phi_2]\!]_{\mathcal{M}} \},$

(1) material: $w \in X \Rightarrow w \in f(w, X)$

$\& \; w \in (W - X) \cup Y \Rightarrow f(w, X) \subseteq Y.$

(2) strict: $f(w, X) = X.$

(3) counterfactual: many versions

(e.g., Kratzer(1981)=SS;Stalnaker(1968)=VCS; Lewis(1973)=VC).

Counterfactuals in SMC

SS=M.P.;RCEA';RCM;RCE;CC+CA+AC+CS+MP (Pollock 1981)

=RCEC;RCK;ID+MOD+CA+CS+MP (Nute & Cross 2002)

$$CA: f(w, X \cup Y) \subseteq f(w, X) \cup f(w, Y)$$

$$AC: f(w, X) \subseteq Y \Rightarrow f(w, X \cap Y) \subseteq f(w, X)$$

$$CS: w \in X \Rightarrow f(w, X) \subseteq \{w\}$$

$$MP: w \in X \Rightarrow w \in f(w, X)$$

$$ID: f(w, X) \subseteq X$$

$$MOD: f(w, W - X) \subseteq X \Rightarrow f(w, Y) \subseteq f(w, X)$$

Standardization of Ordering-Premise Semantics

$$w \in [\![\phi_1 > \phi_2]\!]_{\mathcal{K}}$$

$$\Leftrightarrow \forall u \in \bigcap(\mathcal{B}(w) \cup \{X\}) \exists v \in \bigcap(\mathcal{B}(w) \cup \{X\})$$

$$\forall z \in \bigcap(\mathcal{B}(w) \cup \{X\}).z \leq_{\mathcal{O}(w)} v \,\&\, v \leq_{\mathcal{O}(w)} u \Rightarrow z \in Y$$

$$\Leftrightarrow \forall u \in X \exists v \in X \forall z \in X.z \leq_{\mathcal{O}(w)} v \,\&\, v \leq_{\mathcal{O}(w)} u \Rightarrow z \in Y,$$

where $\forall w. \bigcap \mathcal{O}(w) = \{w\}$ and $X = [\![\phi_1]\!]_{\mathcal{K}}, Y = [\![\phi_2]\!]_{\mathcal{K}[\mathcal{B}'/\mathcal{B}]}$.

$$f_{\mathcal{K}}(w, X) = \{z \in X \mid \forall u \in X \exists v \in X.z \leq_{\mathcal{O}(w)} v \,\&\, v \leq_{\mathcal{O}(w)} u\}$$

Standardization of Ordering-Premise Semantics

$f_{\mathcal{K}}$'s properties

ID : obvious

MP : yes $\because \forall u \in X . w \leq_{\mathcal{O}(w)} w \,\&\, w \leq_{\mathcal{O}(w)} u$

$\because P(u) \cap \mathcal{O}(w) \subseteq P(w) \cap \mathcal{O}(w)$

Kratzer's Samaritan Paradox

For each morally accessible world s :

No murder occurs $(\neg p)$

If a murder occurs, the murderer will be given \$100. $(\neg p \vee q)$

1. $\Box(p \rightarrow q) \therefore$ vacluously true

2. $p \rightarrow \Box q \therefore$ vacuously true

3. $p > q$

$\Leftrightarrow \forall u, z \in [\![p]\!]_{\mathcal{K}} . \exists v \in [\![p]\!]_{\mathcal{K}} . z \leq_{\mathcal{O}(w)} v \,\&\, v \leq_{\mathcal{O}(w)} u \Rightarrow z \in [\![q]\!]_{\mathcal{K}[\mathcal{B}'/\mathcal{B}]}$

s is excluded but "no X must morally occur" cannot be said?

Frank's solution to Kratzer's Samaritan Paradox

No murder occurs ($\neg p$)

If a murder occurs, the murderer will be given \$100.($\neg p \vee q$)

$F = Cn(\{\Box r \mid r \in Cn(\{\neg p, \neg p \vee q\})\})$

$H = Cn(\{r \rightarrow q \mid r \in Cn(Cn(Cn(F \cup \{p\}) - \{p\}) \cup !F)\}$

(ignoring variable assignments;

In H, p and $\neg p$ are contracted)

The Frank-Zvolensky Problem

$f_{\mathcal{K}}$ satisfies ID in any interpretation, i.e., $p > p$.

#If teenagers drink, then they must drink.(deontic)

#If I file my taxes, then I must file my taxes.(bouletic)

#If a=b, then it is necessarily a=b.(alethic by Ogata)

$p > p$ is generally bad?

Frank: $X \cup !\{p > p\} = Cn(Cn(X \cup \{p > p\}) - \{p, \neg p\})$

$p, \neg p$ are undefined in $X \cup !\{p > p\}$

But: OK: If Dalai Lama is mad, he must do so. (=he must have his reason. Etc…by Zvolensky(2002)

Multi-Modal Premise Semantics (Basic Ideas)

- Exploiting Belief-Revision Theory, esp., Model Theories of Dynamic Doxastic Logics (Segerberg 2001) etc.
- Exlpoiting Multi-Modal Logic, esp., BDI-Logics (Rao & Georgeff 1996) etc.

Model Theory of DDL (1)

Let W be any nonempty set.

A topology in W is a family $\mathcal{O}$ of subsets of W such as:

(i) $W, \varnothing \in \mathcal{O}$,

(ii) $\bigcup_{Wi \subseteq W}^{i=\infty} W_i \in \mathcal{O}$,

(iii) $\bigcap_{\varnothing \neq Wi \subseteq W}^{\mathrm{i<n}} W_i \in \mathcal{O}$.

$\langle W, \mathcal{O} \rangle$ is called a topological space.

If $X \in \mathcal{O}$, then X is called open.

If X is open, $W - X$ is called closed.

If X is open and closed, it is called clopen.

Model Theory of DDL (2)

A cover $C \subseteq \wp(\wp(W))(\neq \emptyset)$ of a set $X \subseteq W$ is a family s.t. $X \subseteq \bigcup C$.
If every $Y \in C$ is open, C is called an open cover.
If every open cover of $\mathcal{O}$ has a finite subcover,
$\mathcal{O}$ is called compact.
If for any pair of distinct elements of $\mathcal{O}$,
one is an element of a clopen set of which the other is not,
$\mathcal{O}$ is called totally separated.
If $\mathcal{O}$ is compact and totally separated, it is a Stone topology.
Then $\langle W, \mathcal{O} \rangle$ is called a Stone space.

Model Theory of DDL (3)

An onion in $\langle W, \mathcal{O} \rangle$ is a nonempty family $O \subseteq \wp(\wp(W))$
satisfying the following conditions:
(NESTEDNESS) if $X, Y \subseteq O$, $X \subseteq Y$ or $Y \subseteq X$,
(LIMIT) for any clopen set P, if $P \cap \bigcup O \neq \emptyset$,
then there is a least element $X \in O$ s.t. $P \cap Y \neq \emptyset$.
If evey $X \in O$ is closed, O is called a closed onion.

Model Theory of DDL (4)

If an onion O satisfies the following conditions, it is called a Lewis onion:

(AINT) if $\varnothing \neq C \subseteq O$ then $\bigcap C \in O$;

(AUNI) if $\varnothing \neq C \subseteq O$ then $\bigcup C \in O$;

The belief set B_O of O and the commitment set C_O of O are defined by the conditions:

$B_O = \bigcap O,\ C_O = \bigcup O.$

Model Theory of DDL (5)

Let $\langle W, \mathcal{O} \rangle$ be a Stone space.

An onion frame is a triple $\langle W, \mathcal{O}, D \rangle$

where $D : \wp(W) \rightarrow \wp(\wp(W))$,

called the onion determiner, is defined by the conditions:

(i) for every $X \in dom(D), D(X)$ is an onion,

(ii) $\varnothing \in dom(D)$,

(iii)$\exists C \subseteq W$ s.t. $\forall X \in dom(D), C = \bigcup D(X)$,

(iv) if $X \in dom(D)$, then,

For each clopen set $P. C \cap P \neq \varnothing \Rightarrow S \cap P \in dom(D)$,

where S is the smallest element in $D(X)$ to overlap with P.

$C \in rng(D)$ is called D-possible belief set.

Multi-Modal DDL (1)

Let A be a set of agents and M a set of modalities.

A multimodal index set $Ind(A,M)$ over A and M is defined by:

$\iota ::= m(a) \mid \iota_1 ; \iota_2 \mid \iota_1 + \iota_2 \mid \iota^*$

Multi-Modal DDL (2)

Let $(p \in)\Phi$ be a set of propositional variables

$(w \in)W$ a set of possible worlds,

$(\iota \in)Ind(A,M)$ a multimodal index set,

and $\mathcal{F} = \langle W, \mathcal{O}, D, O, Ind(A,M) \rangle$ a multimodal onion frame.

where $D : Ind(A,M) \times W \to (\wp(W) \to \wp(\wp(W)))$,

and $D(\iota, w)$ is a onion determiner and

$O : Ind(A,M) \times W \to \wp(\wp(W))$,

$O(\iota, w)$ is a onion for each $\iota \in Ind(A,M), w \in W$.

Multi-Modal DDL (3)

Let Φ $(\ni p)$ be a set of propositional variables
and $IndVar(\ni x)$ a variable of multimodal indices.
The language $\mathcal{L}_{IndVar,>,\Phi}$ $(\ni \varphi)$ generated
by Φ and $Ind(A,M)$ is defined by:

$$\varphi ::= p \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \Box_x \varphi \mid \varphi_1 >_x \varphi_2$$

Multi-Modal DDL (4)

Let Φ $(\ni p)$ be a set of propositional variables,
$Ind(A,M)(\ni \imath)$ a multimodal index set,
$\mathcal{L}_{Ind(A,M),>,\Phi}$ $(\ni \varphi)$ a language
$W(\ni w)$ a set of possible worlds, $V : \Phi \to \wp(W)$ a valuation,
$f : W \times \wp(W) \to \wp(W)$ a selection function,
and $\langle W, \mathcal{O}, D, O, Ind(A,M) \rangle$ a multimodal onion frame.

The relation $\langle W, O, f, V, \vec{\imath}, w \rangle \vDash \varphi$ is defined by recursion on φ:

Multi-Modal DDL (5)

$$\langle W,O,f,V,\vec{\imath},w\rangle \vDash p \Leftrightarrow w \in V(p)$$

$$\langle W,O,f,V,\vec{\imath},w\rangle \vDash \varphi_1 \wedge \varphi_2$$

$$\Leftrightarrow \langle W,O,f,V,\vec{\imath},w\rangle \vDash \varphi_1 \;\&\; \langle D,O,f,V,\vec{\imath},w\rangle \vDash \varphi_2$$

$$\langle W,O,f,V,\vec{\imath},w\rangle \vDash \neg\varphi \Leftrightarrow \langle W,O,f,V,\vec{\imath},w\rangle \nvDash \varphi$$

$$\langle W,O,f,V,\vec{\imath},w\rangle \vDash \Box_x \varphi \Leftrightarrow \bigcap O(\vec{\imath}(x),w) \subseteq [\![\varphi]\!]$$

$$[\![\varphi]\!] = \{w \in W \mid \langle W,O,f,V,\vec{\imath},w\rangle \vDash \varphi\}$$

Multi-Modal DDL (6)

Belief-Desire Consistency (cf. BDI-logic)

$$mc(\iota) \in Belief(a), mc(\kappa) \in Desire(a) \Rightarrow (\Box_\iota \varphi \to \neg\Box_\kappa \neg\varphi)$$

$$mc(\kappa) \in Belief(a), mc(\iota) \in Desire(a) \Rightarrow (\Box_\iota \varphi \to \neg\Box_\kappa \neg\varphi)$$

$$mc(\iota) \in Belief(a), mc(\kappa) \in Desire(a) \Rightarrow (\psi >_\iota \varphi \to \neg(\psi >_\kappa \neg\varphi))$$

$$mc(\kappa) \in Belief(a), mc(\iota) \in Desire(a) \Rightarrow (\psi >_\iota \varphi \to \neg(\psi >_\kappa \neg\varphi))$$

Solution to Many Judges Problem

$mc(\vec{\imath}(x)) \in obligatory(a) \Rightarrow \Box_x(\varphi \to \psi) \to (\Box_x \varphi \to \Box_x \psi)$

$mc(\vec{\imath}(x)) \in obligatory(a) \Rightarrow \Box_x \varphi \to \neg\Box_x \neg\varphi$

$mc(\vec{\imath}(x)) \in obligatory(a) \Rightarrow \Box_x \top$

Judge A: $\Box_x liable, \Box_x murderIsACrime, mc(\vec{\imath}(x)) \in obligatory(a)$

Judge B: $\Box_y \neg liable, \Box_y murderIsACrime, mc(\vec{\imath}(y)) \in obligatory(a)$

∴ No Inconsistency

Solution to the Practical Inference Problem

I want not to go to pub: $mc(\vec{\imath}(y)) \in Desire(a) \,\&\, \Box_y \neg pub$

I want to be popular: $mc(\vec{\imath}(y)) \in Desire(a) \,\&\, \Box_y\, pop$

It is the case that if I go to pub, then I will be popular:

$pub >_x pop$

$mc(\vec{\imath}(x)) \in Belief(a) \Rightarrow \neg(pub >_{Desire(a)} \neg pop)$

∴ No Inconsistency

Solution to Kratzer's Samaritan Paradox

No murder occurs: $\Box_{Obligatory(a)} \neg p$

If a murder occurs, the jurors must convene: $p >_{Obligatory(a)} q \nleftrightarrow (\neg p \vee q)$

But #If a murder occurs, the murderer will be given $100.

Other index types:

$Obligation(a), Expectation(a), Predict(a), \ldots$

A Solution to the Frank-Zvolensky Problem

$\langle D, O, V, f, \vec{\imath}, w \rangle \vDash \varphi_1 >_x \varphi_2$

$$\Leftrightarrow \begin{cases} \langle D, O[D(\vec{\imath}(x), w)(Z \cap f(w, [\![\varphi_1]\!])) / (\vec{\imath}(x), w)], V, \vec{\imath}, w \rangle \vDash \varphi_2 \\ \qquad \text{if } Z = \min\{S \in O(\vec{\imath}(x), w) \mid S \cap f(w, [\![\varphi_1]\!]) \neq \varnothing\} \neq \varnothing \\ \langle D, O[D(\imath, w)(\varnothing) / (\vec{\imath}(x), w)], V, \vec{\imath}, w \rangle \vDash \varphi_2 \qquad \text{otherwise} \end{cases}$$

means that $\varphi_1 >_x \varphi_2$ is equivalent to

$[*\bigcirc_\imath \varphi_1]\varphi_2$ where $\bigcirc \in \Box, \Diamond, \ldots$

If f does not satisfy (id) $f(w, X) \subseteq X$,

the logic does not satisfy $\varphi > \varphi$.

Lie $\therefore$ Segerberg(2001) defines selector function semantics of $[*\varphi_1]\varphi_2$

A Solution to the Frank-Zvolensky Problems

$$\langle W, O, f, V, \vec{\imath}, w\rangle \vDash \varphi_1 > \varphi_2$$

$$\Leftrightarrow \begin{cases} f(w, \bigcap(D(\vec{\imath}(x), w)(Z \cap [\![\varphi_1]\!]))) \subseteq [\![\varphi_2]\!] \\ \qquad\qquad \text{if } Z = \min\{S \in O(\vec{\imath}(x), w) \mid S \cap [\![\varphi_1]\!] \neq \varnothing\} \neq \varnothing \\ \langle W, O[D(\vec{\imath}(x), w)(\varnothing)/(\imath, w)], f, V, \vec{\imath}, w\rangle \vDash \varphi_2 \qquad \text{otherwise} \end{cases}$$

and f does not satisfy (id).

Loose End

- I have reviewed Kratzer's Ordering Premise Semantics of Modalities and Conditionals: but the properties of *fK* has still been clarified.
- I have proposed "multimodal" premise semantics based on Dynamic Doxastic Logic, BDI-logics, and Conditional Logics
- I have provided another solution for each problems which are motivations of Ordering-Premise Semantics

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