

Parsing Abstract Categorical Grammars: Complexity and Algorithms

Sylvain Salvati

INPL-INRIA

Introduction

Parsing Abstract Categorical Grammars ([de Groote-2001]):

- The general case
- Natural language case
- Grammars whose abstract language is regular ($\mathcal{L}(2, m)$)

Preliminaries

Given an ACG $\mathcal{G} = \langle \Sigma_1, \Sigma_2, \mathcal{L}, S \rangle$

- \mathcal{G} is lexicalized iff for all abstract constant c , $\mathcal{L}(c)$ contains at least an object constant
- \mathcal{G} is semi-lexicalized iff all abstract constant c either has a second order type or $\mathcal{L}(c)$ contains at least an object constant

The general case

Given an ACG $\mathcal{G} = \langle \Sigma_1, \Sigma_2, \mathcal{L}, S \rangle$ and a term u , question is whether there is an abstract term t of type S so that $\mathcal{L}(t) =_{\beta\eta} u$.

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the item $\langle \Gamma; v; \alpha \rangle$ represents the problem of finding an abstract term t such that:

$$\Gamma \vdash t : \alpha \text{ and } \mathcal{L}(t) =_{\beta\eta} v$$

Rules of General Algorithm

The algorithm is described by a rewriting system on sets of items:

$$\mathcal{I} \rightarrow_A (\mathcal{I} \setminus I) \cup \mathcal{J} \text{ if } I \rightarrow_a \mathcal{J}$$

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- $\langle \Gamma, v, A \rangle \rightarrow_a \{ \langle \Gamma_1; t_1; \alpha_1 \rangle; \dots; \langle \Gamma_n; t_n; \alpha_n \rangle \}$ if there is an abstract constant c of type $\alpha_1 \multimap \dots \multimap \alpha_n \multimap A$ so that $\mathcal{L}(c)t_1 \dots t_n =_{\beta\eta} v$

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To make the last rule work we need solve matching equations (NP-complete)

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 - exponential in the size of the grammar for ACGs coding CFGs

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- Based on an approach proposed by Morrill to parse in Lambek's Calculus [Morrill-2000] and on a way to build proof-nets proposed by de Groote [de Groote-2000].
- It handles polarized items of the form $\langle t, u, A^\epsilon \rangle$ where $\epsilon \in \{+; -; \circ\}$

Incremental algorithm

The use of this algorithm necessitates to prepare the lexicon. Each abstract constant c will be represented by a collection of items $I_c = It(c, \tau(c)^-)$ where:

- $It(t, (\alpha \multimap \beta)^+) = It(x, \alpha^-) \cup It(tx, \beta^+)$ where x is a fresh variable

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Note: each time we use the set of items I_c , we require freshness of the unknowns and the variables it uses.

Handeling items

$\langle t, \mathcal{L}(t), A^+ \rangle$ and $\langle v, \mathcal{L}(v), A^- \rangle$ are complementary if:

- $t = \mathbf{X}x_1 \dots x_n$ and $\{x_1; \dots; x_n\} \subseteq FV(v)$
- $\lambda x_1 \dots x_n.v$ is their *unifier* and \mathbf{X} is their *unification support*

Handling object constants

- When we parse strings we put the constants it contains into a list L . If

$$u = \lambda x.a_1(a_2(\dots(a_n x)\dots)) \quad L_u = [a_1; \dots; a_n].$$

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- $L' \sqsubseteq L$ if $L' \subseteq L$ and the first elements of L is in L' .
- If $L' \subseteq L$ then $L \setminus L'$ is the list where we suppress the top-most elements of L' which are present in L .

Rules of the algorithm

The algorithm acts on triples (\mathcal{I}, L, u) where \mathcal{I} is a set of items and L is a list of object constants.

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- $(\mathcal{I} \cup \{I_1; I_2\}, L) \rightarrow_{Inc} (\mathcal{I}[\mathbf{X} := v] \cup \{\langle v, \mathcal{L}(v), A^\circ \rangle\}, L)$ if I_1 and I_2 are complementary and v is their unifier and \mathbf{X} their unification support

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$(\langle \mathbf{X}, \mathbf{X}, S^+ \rangle, L_u, u) \xrightarrow{*}_{Inc} (\mathcal{I}, [], u)$ with $\langle t, u, S^\circ \rangle \in \mathcal{I}$ iff t is an abstract term of type S and $\mathcal{L}(t) =_{\beta\eta} u$.

Example

- $J := \lambda z. | \text{John} | z : NP$
- $M := \lambda z. | \text{Mary} | z : NP$
- $L := \lambda xyz. x(| \text{loves} | (y z)) : NP \multimap NP \multimap S$

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- $\langle J, \lambda z. |\text{John}| z, NP^- \rangle$
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- $\langle L \mathbf{Y} \mathbf{Z}, \lambda z. \mathbf{Y} (| \text{loves} | (\mathbf{Z} z)), S^- \rangle, \langle \mathbf{Y}, \mathbf{Y}, NP^+ \rangle, \langle \mathbf{Z}, \mathbf{Z}, NP^+ \rangle$

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$\langle J, \lambda z. | \text{John} | z, NP^0 \rangle$

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Example

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- $\langle M, \lambda z. |Mary| z, NP^- \rangle$
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Complexity increases when parsing such phenomena as *garden path*, *left to right quantifier scope*, *centre embedding*... ([Morrill-2000])

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Parsing in $\mathcal{L}(2, m)$

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- In order to remain simple we restrict ourselves to the case where the object signature is of the second order (object terms are trees).

Syntactic descriptions

Syntactic descriptions are the mathematic abstraction guiding the algorithm:

$$\mathcal{D} ::= \{T : T_{\circ}\} \mid \mathcal{D} \multimap \mathcal{D}$$

$[d]$ is the semantics of d :

- $[\{t : \alpha\}] = \{v \mid \cdot \vdash v : \alpha \wedge v =_{\beta\eta} t\}$
- $[d_1 \multimap d_2] = \{v \mid \forall w \in [d_1].(vw) \in [d_2]\}$

a description d is complete if:

- $d = \{t : \alpha\}$ and t is in β -normal η -long form and α is atomic
- $d = d_1 \multimap d_2$ and d_1 and d_2 are complete

Syntactic descriptions: examples

Syntactic descriptions enables to model higher-order contexts:
Given a string $a_1(\dots(a_n(e))\dots)$, the description:

$$\{a_j(\dots(a_n e)\dots) : *\} \multimap \{a_i(\dots(a_n e)\dots) : *\}$$

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Descriptions model the indices involved in Earley parsing

Sequent for parsing

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where:

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- Δ is a context which associates descriptions to variable

Formal system for parsing

$$\frac{\cdot \vdash_L t : \alpha}{\cdot \vdash_A \{t : \alpha\} : \text{Type}} \quad \frac{\cdot \vdash_A d_1 : \text{Type} \quad \cdot \vdash_A d_2 : \text{Type}}{\cdot \vdash_A d_1 \multimap d_2 : \text{Type}} \quad \frac{\cdot \vdash_A d : \text{Type}}{\cdot; x : d \vdash_A x : d}$$

$$\frac{\Gamma_1; \Delta_1 \vdash_A t_1 : \{v_1 : \alpha \multimap \beta\} \quad \Gamma_2; \Delta_2 \vdash_A t_2 : \{v_2 : \alpha\}}{\Gamma_1, \Gamma_2; \Delta_1, \Delta_2 \vdash_A t_1 t_2 \{v_1 v_2 : \beta\}}$$

$$\frac{x_1 : A_1, \dots, x_n : A_n; \cdot \vdash_A t : d \quad \lambda x_1 \dots x_n. t \in \mathcal{L}^{A_1 \multimap \dots \multimap A_n \multimap A}}{x : A; \cdot \vdash x : d}$$

$$\frac{\Gamma; \Delta, x : d_1 \vdash_A t : d_2}{\Gamma; \Delta \vdash_A \lambda x. t : d_1 \multimap d_2} \quad \frac{\Gamma_1; \Delta_1 \vdash_A t_1 : d_1 \multimap d_2 \quad \Gamma_2; \Delta_2 \vdash_A t_2 : d_1}{\Gamma_1, \Gamma_2; \Delta_1, \Delta_2 \vdash_A t_1 t_2 : d_2}$$

Property of the system

If t has an atomic type and is in β -normal η -long form and all the descriptions are complete, then the sequent

$$x_1 : A_1, \dots, x_n : A_n; y_1 : d_1, \dots, y_n : d_n \vdash_A t : \{u : *\}$$

is derivable iff:

there are abstract terms $(t_i)_{i \in [1, n]}$ such that t_i has type A_i
for all terms $(v_i)_{i \in [1, n]}$ such that $v_i \in [d_i]$,

$$t[x_1 := \mathcal{L}(t_1); \dots; x_n := \mathcal{L}(t_n); y_1 := v_1; \dots; y_n := v_n] \in [\{u : *\}]$$

Parsing principle

To obtain the algorithm we use:

- a chart of items
- the items which represent sequents $\Gamma; \Delta \vdash_A t : \{v : o\}$ where t is the subterm of a lexical entry
- rules which emulate the formal system

N.B: this algorithm can easily be extended all the grammars of $\mathcal{L}(2, m)$

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But it is not efficient
- incremental parsing suits well to natural language
- efficient parsing of grammars of $\mathcal{L}(2, m)$ is possible
- try to shift the technology of description for parsing any ACG (proving and matching collaborate to parsing)

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