

5/18 Mathematical Linguistics

an example showing inefficiency of bottom-up stack-based recognizer (Exercise 3.2)

dynamic programming and memoization
Fibonacci sequence

```
int fib(int n)
{
    if (n < 2) {
        return n;
    } else {
        return (fib(n - 1) + fib(n - 2));
    }
}
```

```
int fibDynamic(int n)
{
    int fib[] = new int[n + 1];

    fib[0] = 0;
    fib[1] = 1;
    for (int i = 2; i < n + 1; i++) {
        fib[i] = fib[i - 1] + fib[i - 2];
    }
    return fib[n];
}
```

```
int fibMemoization(int n)
{
    int fib[] = new int[n + 1];

    fib[0] = 0;
    fib[1] = 1;

    return fibMemo(n);
}
```

```
int fibMemo(int n)
{
    if (n < 2 || fib[n] != 0) {
        return fib[n];
    } else {
        fib[n] = fibMemo(n - 1) + fibMemo(n - 2);
        return fib[n];
    }
}
```

}
}

Datalog representation of an instance of the recognition problem

CFG	Datalog program
$A \rightarrow B C$	$A(i,k) \leftarrow B(i,j), C(j,k)$

input	Database
$a_1 \dots a_n$	$\{ a_1(0,1), \dots, a(n-1,n) \}$

Immediate consequence operator

$Cn(\mathbf{P}, E) = \{ Q \mid P_1, \dots, P_n \in E, Q \leftarrow P_1, \dots, P_n \text{ is an instance of a rule in } \mathbf{P} \}$

$D^1 = D,$

$D^{k+1} = D^k \cup Cn(\mathbf{P}, D^k)$

Iterate until stabilizes (naive bottom-up evaluation)

CYK

precompiling the grammar into Chomsky normal form

bottom-up chart parser

\approx on-the-fly compilation into Chomsky normal form

Pushdown automaton $(Q, \Sigma, \Gamma, \delta, q_I, Z_I, Q_F, Z_F)$

transition: $(q, Z) \xrightarrow{a} (r, \gamma) \quad (a \in \Sigma \cup \{ \epsilon \})$

stateless if $|Q| = 1$

extended PDA

transition: $(\beta, Z) \xrightarrow{a} (r, \gamma)$

top-down recognizer: stateless standard PDA

bottom-up recognizer: stateless extended PDA

left-corner recognizer: stateless extended PDA

extended PDA \rightarrow PDA

standard PDA

transition: $(q, Z) \xrightarrow{a} (r, \gamma) \quad (Z \in \Gamma, a \in \Sigma \cup \{ \epsilon \})$

PDA \rightarrow standard PDA

standard PDA \rightarrow stateless standard PDA

stateless standard PDA \rightarrow CFG

left-corner transform

normal PDA \rightarrow CFG

realizable pair (p, q)

Aho et al.'s (1968) method

Lang's tabulation of PDAs

Homework

Exercise 3.7

Problem 3.4

Problem 3.6