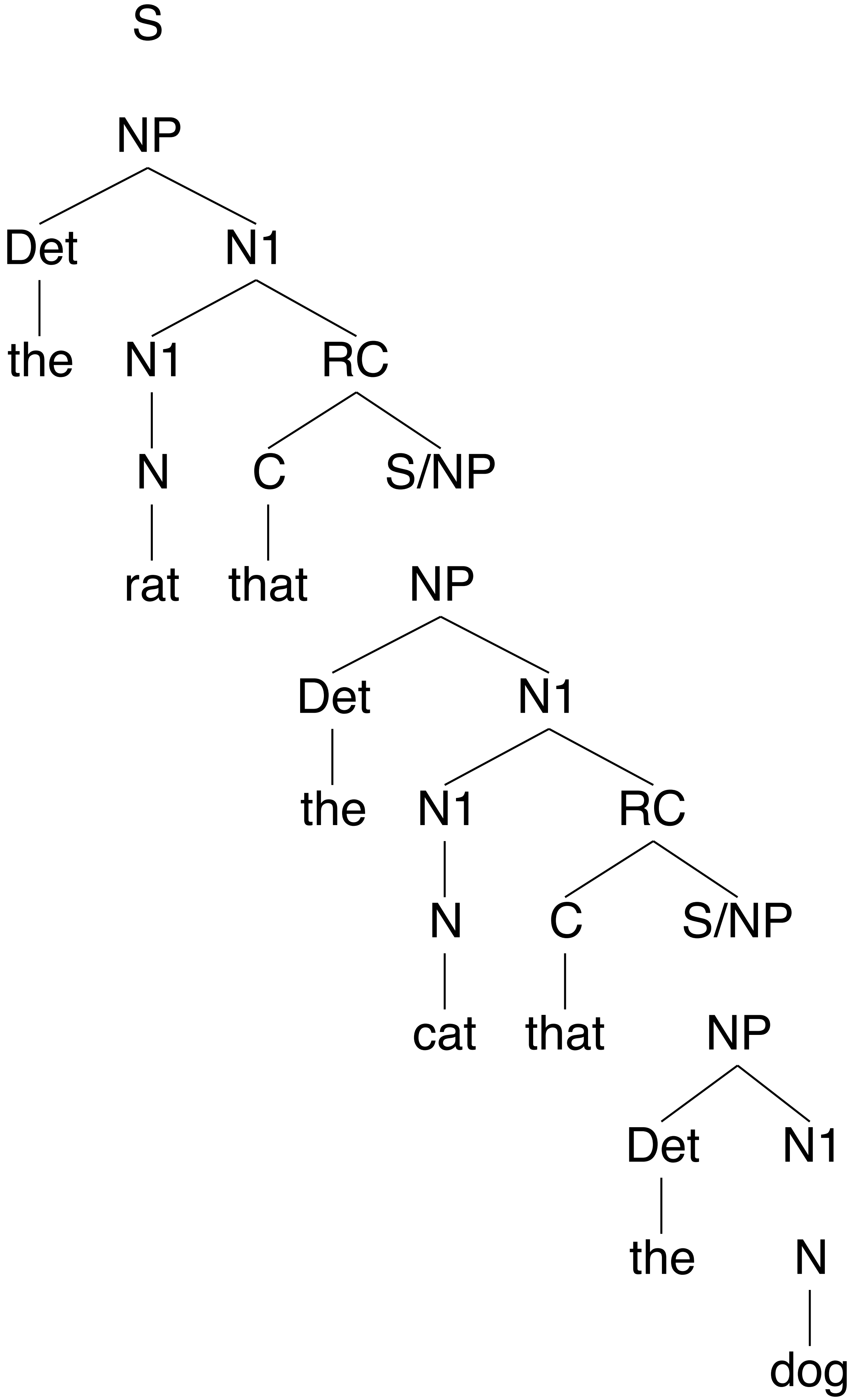
4/27 Mathematical Linguistics

Exercise 2.2, (2.1)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A CFG G is self-embedding

$\iff$ for some nonterminal A and $x, y \in \Sigma^*$,

$$A \overset{*}{\underset{G}{\Rightarrow}} xAy \quad \wedge$$

$$x \neq \varepsilon \quad \wedge \quad y \neq \varepsilon$$

Theorem (Chomsky) Let G be a CFG.

G is not self-embedding $\Rightarrow$ $L(G)$ is regular

Important Properties of the Left-Corner Recognizer

- Shift: $(\alpha, ax) \vdash (a\alpha, x)$ provided $\alpha = \sim X\alpha'$ for some nonterminal X

  $(\varepsilon, ax) \vdash (a, x)$

  $(\sim b\alpha', ax) \vdash (a\sim b\alpha', x)$

  $(X\alpha', ax) \vdash (aX\alpha', x)$

(I) $(\sim Z, x) \vdash^* (\varepsilon, \varepsilon)$ implies $Z \Rightarrow^*_G x$

(II) $(Y\sim X, x) \vdash^* (\varepsilon, \varepsilon)$ implies $X \Rightarrow^+_G Yx$

(III) $(\sim X, x) \vdash^* (\sim Z, \varepsilon)$ implies $X \Rightarrow^*_G xZ$

(IV) $(Y^{\alpha}_{\lambda}, x) \vdash^* (X^{\alpha}_{\lambda}, \varepsilon)$ implies $X \Rightarrow^* Yx$

(V) $(\varepsilon^{\alpha}_{\lambda}, x) \vdash^* (Y^{\alpha}_{\lambda}, \varepsilon)$ implies $Y \Rightarrow^* x$

$(\alpha, x) \vdash^* (\beta, y)$ implies $(\alpha\gamma, xz) \vdash^* (\beta\gamma, yz)$

(I) $(\sim Z, x) \vdash (\varepsilon, \varepsilon)$ (match)

$Z = x = a$ $\quad Z \Rightarrow^*_G x$

$(\sim Z, ax) \vdash (a\sim Z, x)$ (shift)

$\vdash^* (\varepsilon, \varepsilon)$

$Z \Rightarrow^+_G ax$ (by (II))

(II) $(Y\sim X, x) \vdash (\sim Y_2 \cdots \sim Y_n Z\sim X, x)$ (red/pred)

$Z \to YY_2 \cdots Y_n$

$\vdash^* (\sim Y_3 \cdots \sim Y_n Z\sim X, y_3 \cdots y_n z)$

$x = y_2 \cdots y_n z$

$(\sim Y_2, y_2) \vdash^* (\varepsilon, \varepsilon)$

$Y_2 \Rightarrow^* y_2$ by (I)

G: CFG in CNF

$A \to BC$ $\quad X \to Y_1 Y_2$

$A \to a$

$(Y_1\alpha, x) \vdash (\sim Y_2 X\alpha, x)$

$(Y_1 \sim X\alpha, x) \vdash (\sim Y_2\alpha, x)$

$(\sim S, uw) \vdash^* (\sim V X_n \sim Z_n \cdots X_1 \sim Z_1, w) \vdash^* (\varepsilon, \varepsilon)$

$(\sim V_1, x_1 y_1 \cdots x_n y_n v z_n \cdots z_1) \vdash^* (\sim Z_1, y_1 x_2 y_2 \cdots x_n y_n v z_n \cdots z_1)$ $\quad (V_1 = S)$

$\vdash^* (Y_1 \sim Z_1, x_2 y_2 \cdots x_n y_n v z_n \cdots z_1)$

$\vdash (\sim V_2 X_1 \sim Z_1, x_2 y_2 \cdots x_n y_n v z_n \cdots z_1)$ $\quad X_1 \to Y_1 V_2$

$\vdash^* (\sim Z_2 X_1 \sim Z_1, y_2 x_3 y_3 \cdots x_n y_n v z_n \cdots z_1)$

$\vdots$

$\vdash^* (\sim Z_n X_{n-1} \sim Z_{n-1} \cdots X_1 \sim Z_1, y_n v z_n \cdots z_1)$

$\vdash^* (Y_n \sim Z_n X_{n-1} \sim Z_{n-1} \cdots X_1 \sim Z_1, v z_n \cdots z_1)$

$\vdash (\sim V_{n+1} X_n \sim Z_n \cdots X_1 \sim Z_1, v z_n \cdots z_1)$ $\quad X_n \to Y_n V_{n+1}$, $V_{n+1} = V$

$\vdash^* (X_n \sim Z_n \cdots X_1 \sim Z_1, z_n \cdots z_1)$

$\vdash^* (X_{n-1} \sim Z_{n-1} \cdots X_1 \sim Z_1, z_{n-1} \cdots z_1)$

$\vdash^* (\varepsilon, \varepsilon)$

$(\sim V_1, x_1) \vdash^* (\sim Z_1, \varepsilon)$

$V_1 \Rightarrow^*_G x_1 Z_1$

$(\varepsilon, y_1) \vdash^* (Y_1, \varepsilon)$

$Y_1 \Rightarrow^*_G y_1$ $\quad (y_1 \neq \varepsilon)$

$V_2 \Rightarrow^* x_2 Z_2$

$(\sim V_{n+1}, v) \vdash^* (\varepsilon, \varepsilon)$

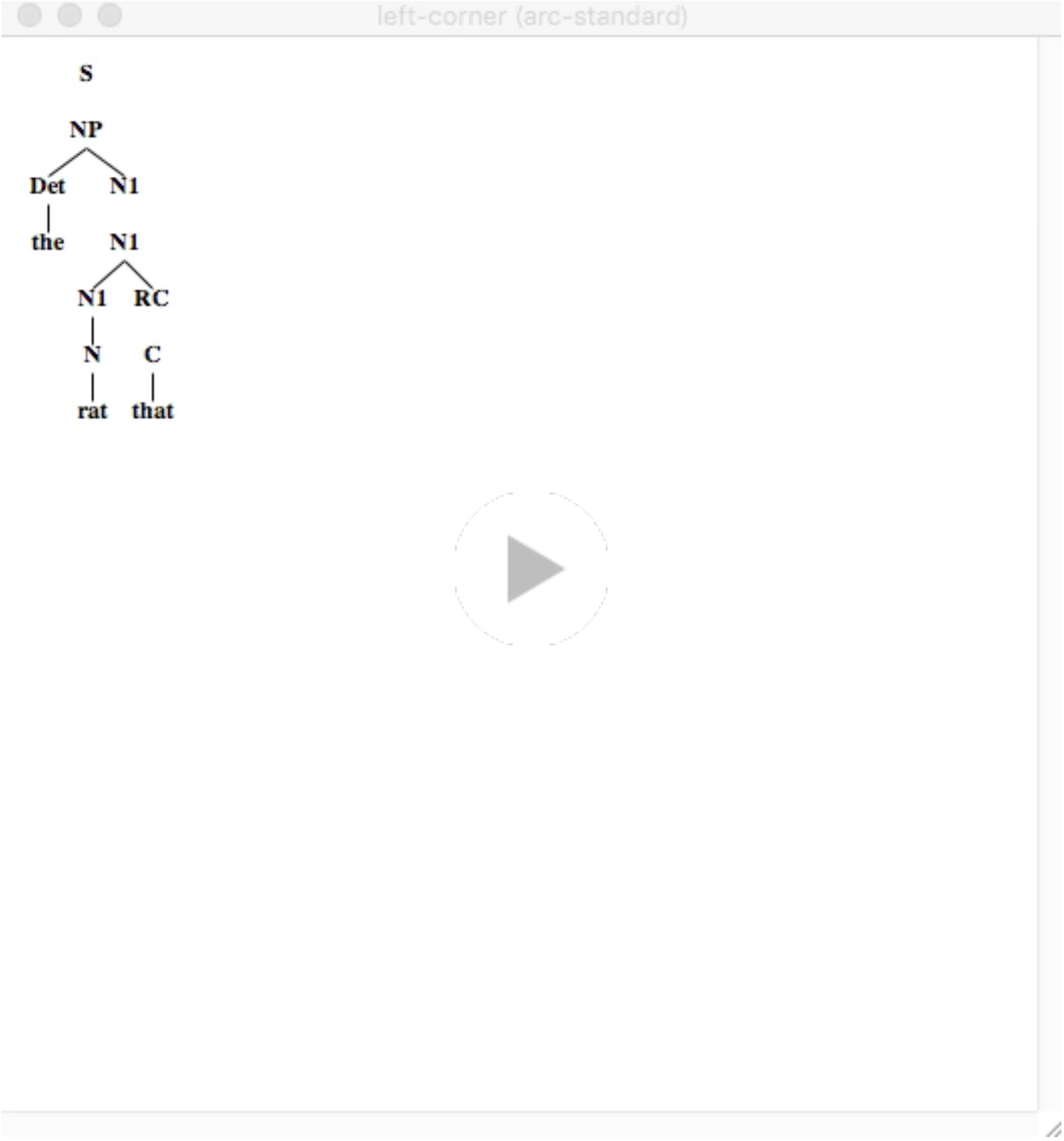
$V_{n+1} \Rightarrow^*_G v$

$(X_n \sim Z_n, z_n) \vdash^* (\varepsilon, \varepsilon)$

$Z_n \Rightarrow^+_G X_n z_n$ $\quad (z_n \neq \varepsilon)$

Exercise 2.6.

(2.1)



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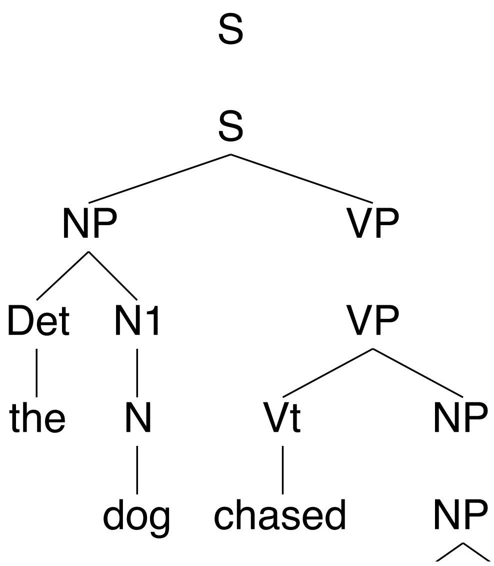

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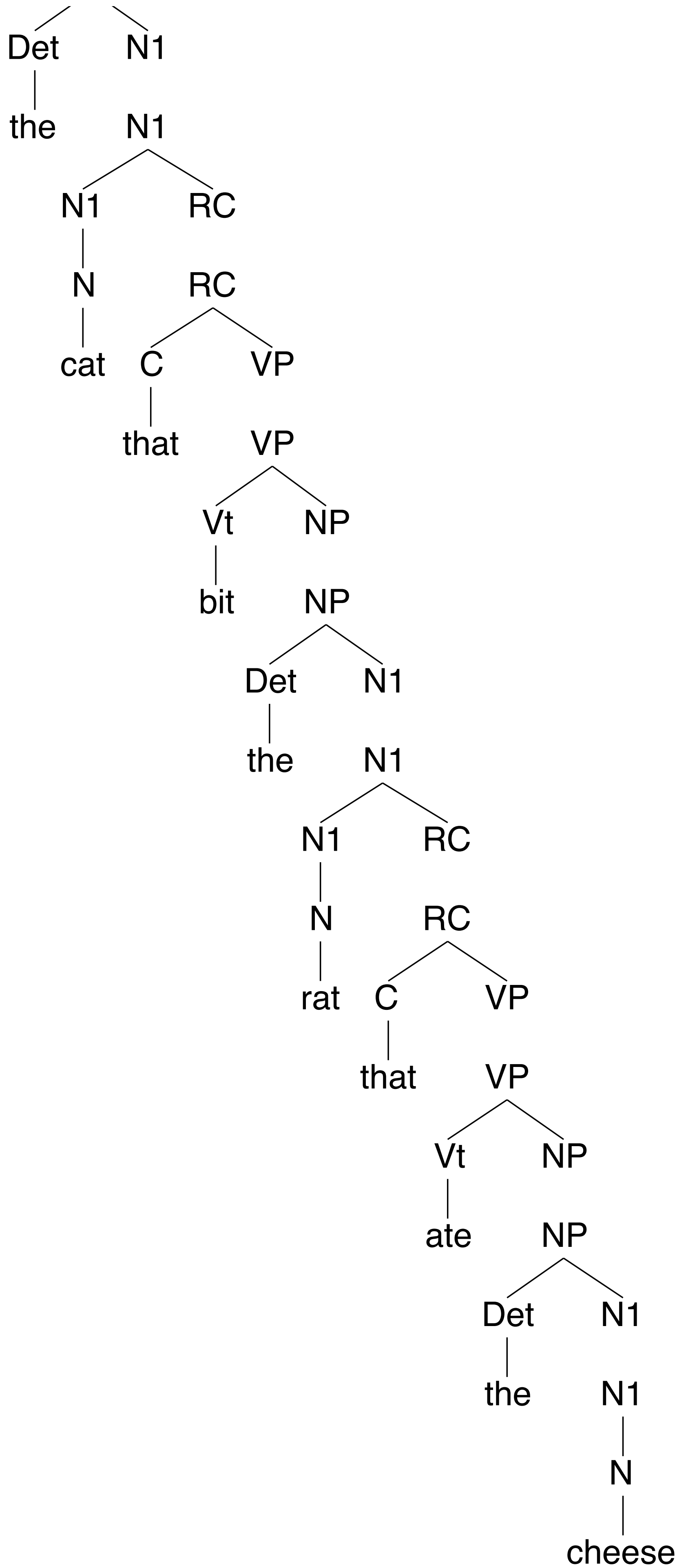
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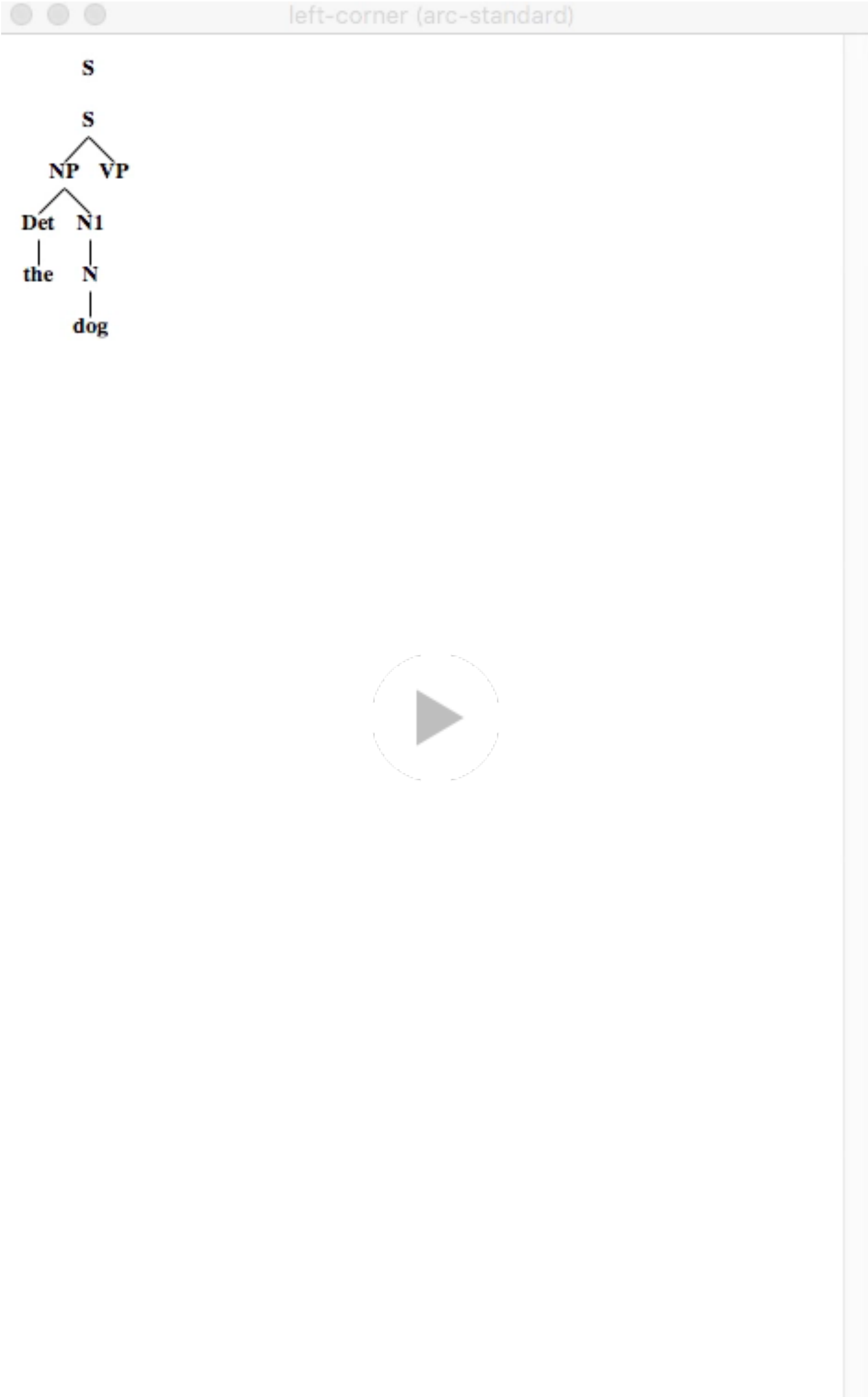
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